Proceedings of the Second Asian Conference on Mechanics of Functional Materials and Structures
October 22-25, 2010, Nanjing, China

SEVERAL CENTRAL INTERFACE CRACK SOLUTION UNDER ARBITRARY MATERIAL COMBINATION

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This paper deals with a central interface crack in a bonded finite plate and periodic interface cracks. Then, the effects of material combination and relative crack length on the stress intensity factors are discussed. A useful method to calculate the stress intensity factor of interface crack is presented with focusing on the stress at the crack tip calculated by the finite element method. For periodic interface cracks, it is found that the stress intensity factors are only depending on bimaterial parameter ε and increase with increasing ε and the relative crack length *a/W*. For a central interface crack, it is found that the stress intensity factors are depending on the Dunders' parameters α and β. The variation of dimensionless stress intensity factors F_I, F_{II} are discussed under arbitrary material combinations with varying relative crack length *a/W*.

Key words: *Central Interface Crack, Periodic Interface Crack, Bonded Finite Plate, Stress Intensity Factor, Finite Element Method*

1 Introduction

For the discussion of an interface crack in a bonded finite plate, although a lot of related studies were published previously, few solutions are available under arbitrary material combinations. In this paper, therefore, periodic interface cracks as shown in Fig.1 (a) will be treated in comparison with a central interface crack in bonded finite plates as shown in Fig.1 (b). Then, the effects of relative crack length on the stress intensity factors will be analyzed explicitly under arbitrary material combination. In Fig.1 (a), along $x=(1+2n)W$ (n is the integer) the boundary conditions are $u_x = 0$, $\tau_{xy} = 0$ but $\sigma_x \neq 0$. On the other hand, in Fig.1 (b) along $x=\pm W$, the boundary conditions are $\sigma_x = 0$, $\tau_{xy} = 0$.

2 Analysis Method

The analysis method used in this research is based on the stresses at the crack tip calculated by FEM. For homogenous material, stress intensity factors can be obtained with a good accuracy by using the proportional stress fields for the reference and given problems [2, 3].

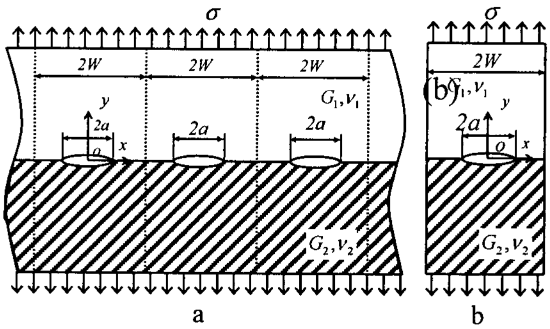

Figure 1: (a) Periodic interface cracks in an infinite bonded plate and (b) a central interface crack in a bonded plate.

In this paper, a useful method to calculate the stress intensity factor of interface crack is presented [1].

An effective method was recently proposed by Oda et al. successfully to analyze interface crack problems [1]. It is well known that there exists oscillation singularity at the interface crack tip. From the stresses σ_y, τ_{xy} along the interface crack tip, stress intensity factors are defined as

$$\sigma_y + i\tau_{xy} = \frac{K_I + iK_{II}}{\sqrt{2\pi r}}\left(\frac{r}{2a}\right)^{i\varepsilon}, \quad r \to 0, \qquad (1)$$

From Eq.(1), it is known that because stress intensity factors for interface crack and the crack in homogenous material are different, it is difficult to separate modes

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absolutely. So it is necessary to obtain the following equation from Eq.(1)

$$K_I = \lim_{r\to 0}\sqrt{2\pi r}\sigma_y\left(\cos Q + \frac{\tau_{xy}}{\sigma_y}\sin Q\right), \quad (2)$$

$$K_{II} = \lim_{r\to 0}\sqrt{2\pi r}\tau_{xy}\left(\cos Q + \frac{\sigma_y}{\tau_{xy}}\sin Q\right), \quad (3)$$

$$Q = \varepsilon \ln(\frac{r}{2a}). \quad (4)$$

If the distance r is given as a constant, the following equation can be obtained.

$$Q^* = Q, \quad \frac{\tau_{xy}{}^*}{\sigma_y{}^*} = \frac{\tau_{xy}}{\sigma_y} \quad (5)$$

Here, values without (*) are for unknown problem and values with (*) are for reference problem.

Therefore if Eq. (5) is satisfied, Eq.(6) may be derived from Eq.(2) and Eq.(3). In such case, oscillatory items of the reference and unknown problems become the same:

$$\frac{K_I{}^*}{\sigma_y{}^*} = \frac{K_I}{\sigma_y}, \quad \frac{K_{II}{}^*}{\tau_{xy}{}^*} = \frac{K_{II}}{\tau_{xy}} \quad (6)$$

Here, $\sigma^*_{y0,FEM}$, $\tau^*_{xy0,\mathrm{FEM}}$ are stresses of the reference problem calculated by FEM, and $\sigma_{y0,\mathrm{FEM}}$, $\tau_{xy0,\mathrm{FEM}}$ are stresses of the given unknown problem. Stress intensity factors of the given unknown problem can be obtained by:

$$K_I = \frac{\sigma_{y0,FEM}}{\sigma_{y0,FEM}{}^*}K_I{}^* \quad (7)$$

$$K_{II} = \frac{\tau_{y0,FEM}}{\tau_{y0,FEM}{}^*}K_{II}{}^* \quad (8)$$

Stress intensity factors of the reference problem are defined by

$$K_I{}^* + iK_{II}{}^* = (T + iS)\sqrt{\pi a}(1 + 2i\varepsilon) \quad (9)$$

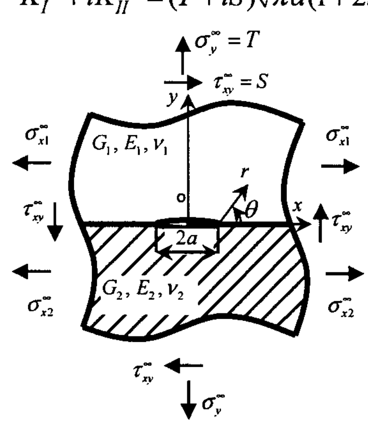


Figure 2: Reference problem ($\varepsilon_{x1} = \varepsilon_{x2}$ at $y = 0$)

Regarding the reference problem in Fig.2, denote $\sigma^{T=1,S=0}_{y0,FEM}{}^*$, $\tau^{T=1,S=0}_{xy0,FEM}{}^*$ are values of stresses for $(T,S) = (1,0)$, and $\sigma^{T=0,S=1}_{y0,FEM}{}^*$ $\tau^{T=0,S=1}_{xy0,FEM}{}^*$ are ones for $(T,S) = (0,1)$. In order to satisfy Eq. (5), stresses at the crack tip of the reference problem are expressed as

$$\begin{aligned}\sigma_{y0,FEM}{}^* &= \sigma^{T=1,S=0}_{y0,FEM}{}^* \times T + \sigma^{T=0,S=1}_{y0,FEM}{}^* \times S,\\ \tau_{xy0,FEM}{}^* &= \tau^{T=1,S=0}_{xy0,FEM}{}^* \times T + \tau^{T=0,S=1}_{xy0,FEM}{}^* \times S\end{aligned} \quad (10)$$

By substituting Eq.(5) into Eq.(10) with T=1, the value of S is obtained as

$$S = \frac{\sigma_{y0,FEM} \times \tau^{T=1,S=0}_{xy0,FEM}{}^* - \tau_{xy0,FEM} \times \sigma^{T=1,S=0}_{y0,FEM}{}^*}{\tau_{xy0,FEM} \times \sigma^{T=0,S=1}_{y0,FEM}{}^* - \sigma_{y0,FEM} \times \tau^{T=0,S=1}_{xy0,FEM}{}^*}. \quad (11)$$

The problem that is subjected to $T = 1$ and S expressed by Eq.(11) is considered as the reference problem. Because the exact solution is known, the error of the unknown problem can be evaluated by using the same mesh. In the following, results are shown using dimensionless stress intensity factors F_I , F_{II} defined by

$$K_I + iK_{II} = (F_I + iF_{II})(1 + 2i\varepsilon)\sigma\sqrt{\pi a}, \quad (12)$$

$$\varepsilon = \frac{1}{2\pi}\ln\left[\left(\frac{\kappa_1}{G_1} + \frac{1}{G_2}\right)\Big/\left(\frac{\kappa_2}{G_2} + \frac{1}{G_1}\right)\right] \quad (13)$$

$$\kappa_m = \begin{cases}(3-\nu_m)/(1+\nu_m) & \text{(Plane stress)}\\ 3-4\nu_m & \text{(Plane strain)}\end{cases}$$
$$\nu_m: \text{ (Poisson's ratio)} \quad (m=1,2)$$
$$G_m: \text{ (Shear modulus)} \quad (m=1,2) \quad (14)$$

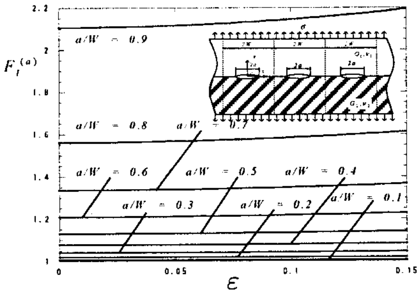
The Dundurs' bi-material parameters α, β are defined as

$$\alpha = \frac{G_1(\kappa_2+1) - G_2(\kappa_1+1)}{G_1(\kappa_2+1) + G_2(\kappa_1+1)}, \quad \beta = \frac{G_1(\kappa_2-1) - G_2(\kappa_1-1)}{G_1(\kappa_2+1) + G_2(\kappa_1+1)} \quad (15)$$

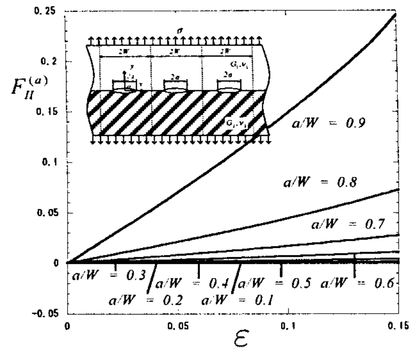
3 Stress Intensity Factors for Periodic Interface Cracks

Periodic interface cracks are one of the most fundamental problems. However, so far as the authors know, the solution is not available under arbitrary material combinations. Similarly to the problem of bonded infinite plate subjected to the internal pressure, the stress intensity factors only depend on the parameter ε .

Figure 3 (a) shows the relation between $F_I{}^{(a)}$ and ε with different a/W . From the figure, it is known that $F_I{}^{(a)}$ slightly increases with increasing ε when a/W is fixed; the increment is larger when a/W is larger. Figure 3 (b) shows the relation between $F_{II}{}^{(a)}$ and ε with different a/W . Similarly to $F_I{}^{(a)}$, the values of $F_{II}{}^{(a)}$ also increases with increasing ε when a/W is fixed, and the increment becomes larger when a/W is larger.



(a)



(b)

Figure 3: The relationship between (a) $F_I^{(a)}$ vs. ε (b) $F_{II}^{(a)}$ vs. ε with different *a/W* in Fig.1 (a)

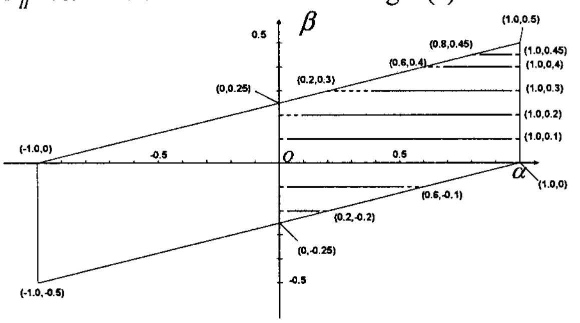


Figure 4: The map of α and β

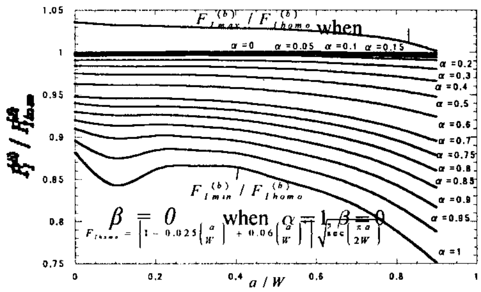
4 Stress Intensity Factors for the Interface Crack in a Bonded Finite Plate

Regarding another fundamental problem in Fig.1 (b), the effects of relative crack length *a/W* on stress intensity factors will be discussed under arbitrary combinations. When material 1 and material 2 are exchanged, Dundur's parameters (α, β) become $(-\alpha, -\beta)$, and then the stress intensity factors (F_I, F_{II}) become (F_I, $-F_{II}$). Therefore all material combinations are considered in the range $\alpha > 0$ as shown in Fig.4. For special material combinations indicated by the dashed lines in Fig.4, calculations cannot be executed by the current finite element method code, and the results for the region are obtained by extrapolation using the results that can be calculated.

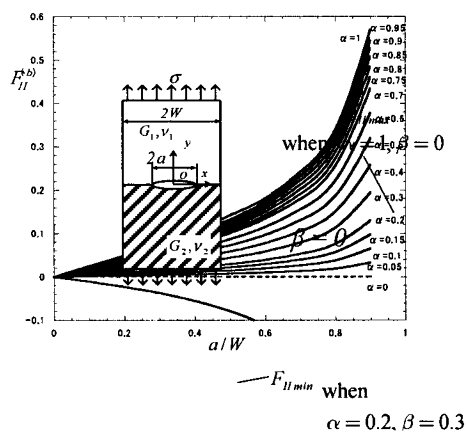
To save space, Fig.5 only shows the variations $F_I^{(b)} / F_{I homo}^{(b)}$ and $F_{II}^{(b)}$ for $\beta = 0$ as an example. Here, $F_{I homo}^{(b)}$ means results for homogenous material, and it has been obtained by Isida [4] and the approximate solutions expressed in formula (16) [5].

$$F_{I homo}^{(b)} = \left\{1-0.025\left(\frac{a}{W}\right)^2+0.06\left(\frac{a}{W}\right)^4\right\}\sqrt{\sec\left(\frac{\pi a}{2W}\right)} \qquad (16)$$

It should be found that the ratio $F_I^{(b)} / F_{I homo}^{(b)}$ largely depends on α and distributes in a wide region as $1.000 \sim 0.751$. The ratio $F_I^{(b)} / F_{I homo}^{(b)}$ is close to 1 when α is small, and becomes smaller when α is large. When $\alpha = 1$, $F_I^{(b)} / F_{I homo}^{(b)}$ takes the minimum value for all regions of a/W. On the other hand, $F_{II}^{(b)}$ increases with increasing α, and the increment becomes larger when the crack length is larger. For almost all regions of crack length, $F_{II}^{(b)}$ takes the maximum when $\alpha = 1$ except the case when the crack length is extremely large.



(a)



(b)

Figure 5: (a) $F_I^{(b)} / F_{I homo}^{(b)}$ vs. a/W and (b) $F_{II}^{(b)}$ vs. a/W when $\beta = 0$.

Considering $F_I^{(b)} / F_{I\,homo}^{(b)}$ when $a/W \leq 0.9$, it is known that $0.751 < F_I^{(b)} / F_{I\,homo}^{(b)} < 1.036$, and mostly it distributes in the region which is a little smaller than 1.

5 Conclusions

In this paper, the stress values at the crack tip calculated by FEM are used and the stress intensity factors of interface cracks are evaluated from the ratio of stress values between a reference problem and a given problem. Then the stress intensity factors are discussed with the following conclusions.

(1) For periodic interface cracks in a bonded plate shown in Fig.1 (a), the effects of relative crack length and material combinations on the stress intensity factors have been discussed. Stress intensity factors $F_I^{(a)}$, $F_{II}^{(a)}$ increase with increasing ε (Fig. 3).

(2) For a central interface crack in a bonded finite plate shown in Fig.1 (b), the effects of relative crack length and material combinations on the stress intensity factors have been discussed. The ratio to the results to the homogeneous material $F_I^{(b)} / F_{I\,homo}^{(b)}$ is in the region $0.751 < F_I^{(b)} / F_{I\,homo}^{(b)} < 1.036$ when $a/W \leq 0.9$, and mostly it distributes in the region which is a little smaller than 1.

Generally, F_I always takes the maximum value when $\alpha = 0.2, \beta = 0.3$ and minimum value when $\alpha = 1.0, \beta = 0$. On the other hand, F_{II} always takes the maximum value when $\alpha = 1.0, \beta = 0$ and minimum value when $\alpha = 0.2, \beta = 0.3$ except the case when the crack length is extremely large.

(3) From the comparison between the results for periodic interface cracks and a central interface crack in a finite bonded plate, it is seen that the results of periodic interface cracks are close to the results of a central interface crack in a bonded finite plate when α is small and the crack length is small. The results of periodic interface cracks are close to the results of a central interface crack in a bonded finite plate when α is large and the crack length is large.

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