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SEVERAL EDGE INTERFACE CRACK SOLUTION UNDER ARBITRARY MATERIAL COMBINATION

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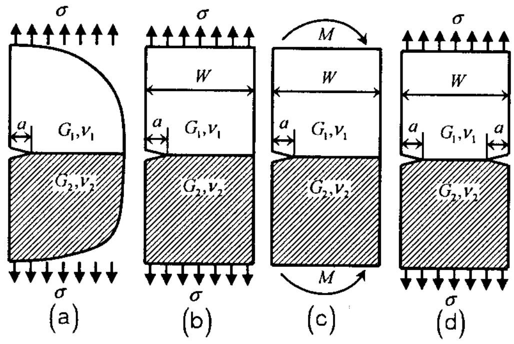
This paper deals with several edge interface cracks in bonded plates subjected to tension and bending moment. Then, the effects of material combination on the stress intensity factors are discussed. The stress intensity factors are indicated in charts under arbitrary material combination. For the edge interface crack under tension, it is found that the dimensionless stress intensity factors are not always finite depending on Dunders' parameters α,β . For example, F_I is infinite when $\alpha(\alpha-2\beta)>0$. And they are finite when $\alpha(\alpha-2\beta)=0$, and zero when $\alpha(\alpha-2\beta)<0$. It is found that for any material combination of $\alpha(\alpha-2\beta)>0$, $\alpha(\alpha-2\beta)=0$ and $\alpha(\alpha-2\beta)<0$, values of $F_I\cdot(a/W)^{1-\lambda}$ and $F_{II}\cdot(a/W)^{1-\lambda}$ are constant independent of a/W when $a/W<10^{-3}$, where a/W is the normalized crack size. As a result, the stress intensity factors F_I,F_{II} can be expressed as $F_I=C_I\times(a/W)^{1-\lambda}$, $F_{II}=C_{II}\times(a/W)^{1-\lambda}$. Similar conclusions can also be given for the edge interface crack under bending. Furthermore, dimensionless stress intensity factors of single and double edge interface cracks in bonded finite plates are also investigated with changing Dunders' material combination ratios and relative crack size a/W .

Keywords: *Elasticity, Fracture Mechanics, Stress Intensity Factor, Edge Interface Crack, Bonded Strip*

1 Introduction

It is widely known that a crack usually initiates at the surface of a homogenous plate, then, gradually grows into inside. Similar phenomenon can be observed for the bonded plate. Edge crack problems in homogenous plates have been systematically investigated and solved. However, for the edge interface crack problem in Fig. 1(b), only those under several given material combinations have been investigated. Furthermore, the problem in Fig. 1(a) has not been solved yet till recently.

In this paper, several edge interface crack problems are investigated systematically with varying material combination as well as the normalized crack size. Firstly, the stress intensity factors of a bonded semi-infinite plate will be totally solved. Then, variations of dimensionless stress intensity factors in the bonded finite plate under tension and bending moment will also be discussed. In the end, single and double edge interface cracks in the bonded finite plates are also investigated and compared.



(a) A bonded semi-infinite plate under remote tension
(b) A bonded finite plate under tension
(c) A bonded finite plate under bending moment
(d) A double edge interface crack in a bonded finite plate under tension

Figure 1: Edge interface crack problems

Recently, an effective method was proposed for calculating the stress intensity factor in homogenous plates [1]. Then, the method is successfully extended to interface crack problems [2]. Both of those methods utilize the stress values at the crack tip by FEM. From

2 The Zero Element Method Extended to the Interface Crack Problems

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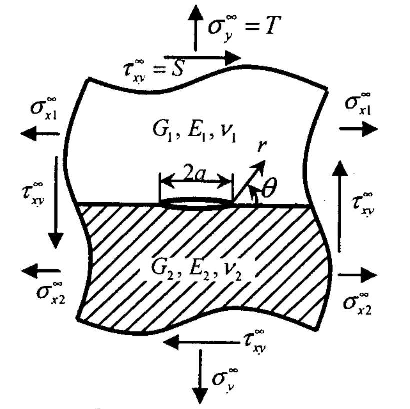
the stresses σ_y, τ_{xy} along the interface crack tip, the stress intensity factors are defined as

$$\sigma_y + i\tau_{xy} = \frac{K_I + iK_{II}}{\sqrt{2\pi r}}\left(\frac{r}{2a}\right)^{i\varepsilon} \quad , \quad r \to 0 \qquad (20)$$

$$\varepsilon = \frac{1}{2\pi}\ln\left[\left(\frac{\kappa_1}{G_1}+\frac{1}{G_2}\right)/\left(\frac{\kappa_2}{G_2}+\frac{1}{G_1}\right)\right], \qquad (2)$$



$$\sigma_{x2}^\infty = \frac{1}{1+\kappa_2}\left[\frac{\mu_2}{\mu_1}(1+\kappa_1)\sigma_{x1}^\infty + \left\{3-\kappa_2-\frac{\mu_2}{\mu_1}(3-\kappa_1)\sigma_y^\infty\right\}\right]$$

Figure 2: Reference problem

From Eq. (1), the stress intensity factors may be separated as:

$$K_I = \lim_{r\to 0}\sqrt{2\pi r}\sigma_y\left(\cos Q + \frac{\tau_{xy}}{\sigma_y}\sin Q\right),$$

$$K_{II} = \lim_{r\to 0}\sqrt{2\pi r}\tau_{xy}\left(\cos Q + \frac{\sigma_y}{\tau_{xy}}\sin Q\right), \qquad (3)$$

$$Q = \varepsilon\ln(\frac{r}{2a})$$

Here, *r* and *Q* can be chosen as constant values since the reference and unknown problems have the same mesh pattern and material combination. Therefore if Eq. (4) is satisfied, Eq. (5) may be derived from Eq. (3).

$$\tau_{xy}^* / \sigma_{xy}^* = \tau_{xy} / \sigma_y \qquad (4)$$

$$K_I*/\sigma_y* = K_I/\sigma_y, \quad K_{II}*/\tau_{xy}* = K_{II}/\tau_{xy} \qquad (5)$$

The stress intensity factors of the given unknown problem can be obtained by:

$$K_I = \frac{\sigma_{y0,FEM}}{\sigma_{y0,FEM}*}K_I*, \quad K_{II} = \frac{\tau_{xy0,FEM}}{\tau_{xy0,FEM}*}K_{II}* \qquad (6)$$

Here, σ_y^*, τ_{xy}^* are stresses of the reference problem calculated by FEM, and σ_y, τ_{xy} are stresses of the given unknown problem. The stress intensity factors of the reference problem are defined by

$$K_I^* + iK_{II}^* = (T+iS)\sqrt{\pi a}(1+2i\varepsilon) \qquad (7)$$

Regarding the reference problem in Fig.2, $\sigma_{y0,FEM}^{T=1,S=0}*$ $\tau_{xy0,FEM}^{T=1,S=0}*$ are stress values for $(T,S)=(1,0)$, and $\sigma_{y0,FEM}^{T=1,S=0}*$ $\tau_{xy0,FEM}^{T=0,S=1}*$ are those for $(T,S)=(0,1)$. In order to satisfy Eq. (5), stresses at the crack tip of the reference problem are expressed as:

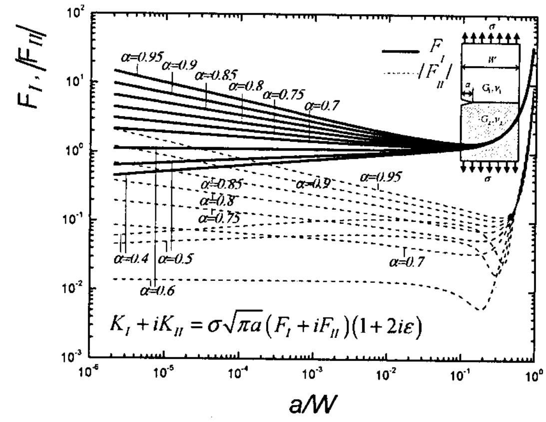


Figure 3: Variations of F_I, F_{II} with a/W

$$\sigma_{y0,FEM}* = \sigma_{y0,FEM}^{T=1,S=0}* \times T + \sigma_{y0,FEM}^{T=0,S=1}* \times S,$$
$$\tau_{xy0,FEM}* = \tau_{xy0,FEM}^{T=1,S=0}* \times T + \tau_{xy0,FEM}^{T=0,S=1}* \times S \qquad (8)$$

If we assume T=1, the value of S can be obtained from Eq. (8) as:

$$S = \frac{\sigma_{y0,FEM} \times \tau_{xy0,FEM}^{T=1,S=0}* - \tau_{xy0,FEM} \times \sigma_{y0,FEM}^{T=1,S=0}*}{\tau_{xy0,FEM} \times \sigma_{y0,FEM}^{T=0,S=1}* - \sigma_{y0,FEM} \times \tau_{xy0,FEM}^{T=0,S=1}*} \qquad (9)$$

3 Analysis Results

In this research, the following dimensionless stress intensity factors F_I, F_{II} are used:

$$K_I + iK_{II} = (F_I + iF_{II})(1+2i\varepsilon)\sigma\sqrt{\pi a} \qquad (10)$$

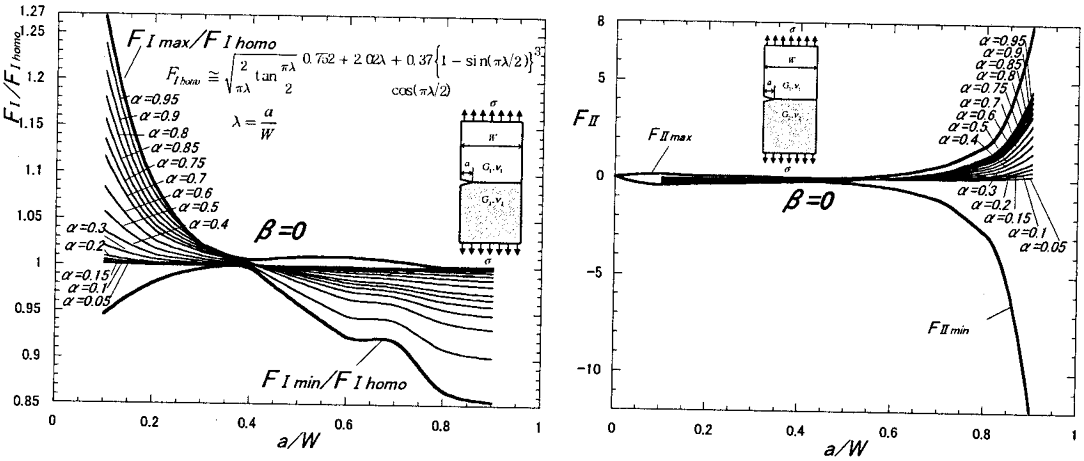
For 2D interface crack, it is known that the stress intensity factor is determined by Dunders' bi-material combination α, β alone. The definitions of α, β are

$$\alpha = \frac{G_1(\kappa_2+1)-G_2(\kappa_1+1)}{G_1(\kappa_2+1)+G_2(\kappa_1+1)}, \quad \beta = \frac{G_1(\kappa_2-1)-G_2(\kappa_1-1)}{G_1(\kappa_2+1)+G_2(\kappa_1+1)} \qquad (11)$$

If the edge crack problems in Fig. 1 were rotated 180° around the midpoint of the interface in the plane of the paper, Dunders' bi-material parameters α, β would change into $-\alpha, -\beta$ according to Eq. (11). However, in this case, the absolute values of the stress intensity factors of Fig. 1 will not vary with this rotation. In fact, the values F_I, F_{II} change into $F_I, -F_{II}$ after the rotation. This leads us to the conclusion that $F_I \to F_I, F_{II} \to -F_{II}$ when $\alpha \to -\alpha$, $\beta \to -\beta$. Therefore, only the stress intensity factors under $\alpha \geq 0$ are shown in this research.

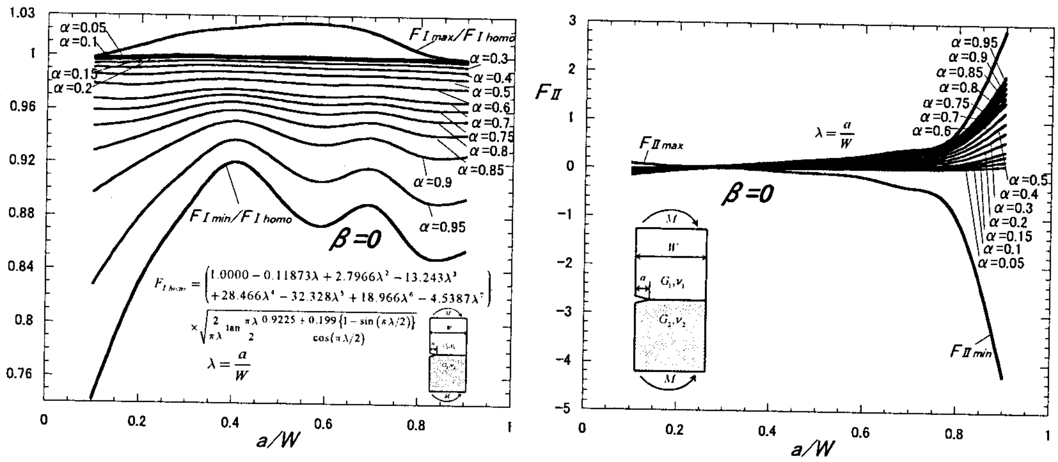
3.1 Stress Intensity Factors of the Edge Interface Crack in a Bonded Semi-Infinite Plate

Figure 3 shows the variations of linear F_I, F_{II} under β=0.3 over the linear normalized crack size a/W from 0.0006 to 0.9 for the problem shown in Fig. 1(b). It can be seen that there is a linear relationship between



(a) Values of $F_I / F_{I\,\mathrm{homo}}$ (b) Values of F_{II}

Figure 4: Stress intensity factors of an edge interface crack in a bonded finite plate under tension



(a) Values of $F_I / F_{I\,\mathrm{homo}}$ (b) Values of F_{II}

Figure 5: Stress intensity factors of an edge interface crack in a bonded plate under bending moment

logarithmic $F_I, |F_{II}|$ and a/W when $a/W < 10^{-3}$. In other words, $F_I \propto (a/W)^S$, $F_{II} \propto (a/W)^S$ can be satisfied when $a/W < 10^{-3}$, where, the value S is a constant which is determined by the singular index of the bonded plate. Similar conclusions can also be made for other material combinations. In conclusion, the following relationship can be found for the semi-infinite plate ($a/W \to 0$)

$$\alpha(\alpha-2\beta) > 0 : F_I, F_{II} \to \infty;$$
$$\alpha(\alpha-2\beta) = 0 : F_I, F_{II} \text{ are finite}; \quad (12)$$
$$\alpha(\alpha-2\beta) < 0 : F_I, F_{II} \to 0$$

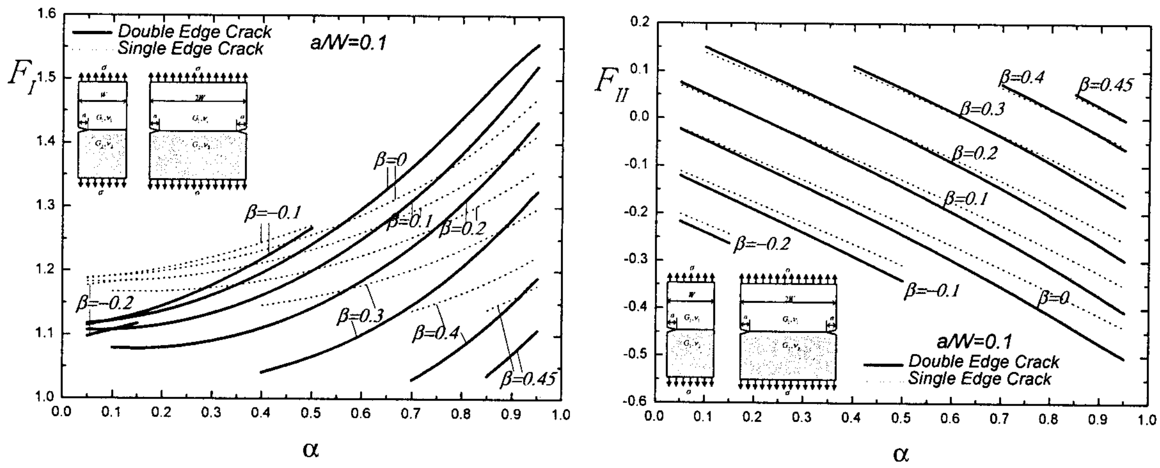
Equation (12) can be explained by the existence of singular stress field in a bonded plate. It is known that the existence of singular stress is controlled by the following relations [3], [4]:

$$\alpha(\alpha-2\beta) > 0 : \lambda < 0, \text{ Zero stress}; \quad (13)$$
$$\alpha(\alpha-2\beta) = 0 : \lambda = 0, \text{ Finite value of stress};$$
$$\alpha(\alpha-2\beta) < 0 : \lambda > 0, \text{ Singular stress.}$$

3.2 Stress Intensity Factors of Single Edge Interface Crack in a Bonded Finite Plate Subjected to Tension And Bending Moment.

Stress intensity factors of an edge interface crack in a bonded plate subjected to tension and bending moment are systematically investigated with varying the normalized crack size $a/W = 0.1 \sim 0.9$.

Figure 4 shows the variations of the stress intensity factors in a bonded plate under tension with varying the normalized crack size a/W when $\beta = 0$. In Fig. 4(a), the value F_I is normalized using the dimensionless stress



(a) F_I of edge interface cracks $a/W = 0.1$ (b) F_{II} of edge interface cracks $a/W = 0.1$

Figure 6: Stress intensity factors of single and double edge interface cracks in bonded plates under tension

intensity factor in a homogenous plate $F_{I\,\mathrm{homo}}$ [5,6]. The curves at the top and bottom of each figure show the maximum and minimum stress intensity factors of all material combination respectively. Others show the stress intensity factors of given α, β. As can be seen from Fig. 4(a), there is a crossing point around a/W=0.4. The ratio $F_I / F_{I\,\mathrm{homo}}$ grows with the increase of α before this point, and grows with the decrease of α after this point. However, F_{II} grow monotonically with the increase of α when β is kept constant.

Figure 5 shows the variations of the stress intensity factors in a bonded plate under bending moment with varying the normalized crack size a/W when $\beta = 0$. Similarly, the curves at the top and bottom of Fig. 5 show the maximum and minimum dimensionless stress intensity factors of all material combination ratios respectively. The ratio $F_I / F_{I\,\mathrm{homo}}$ increases monotonically with the decrease of α when β is kept constant, and F_{II} grows with the increase of α.

3.3 Stress Intensity Factors of the Double Edge Interface crack in a Bonded Plate under Tension.

Figure 6 shows the dimensionless stress intensity factors of single and double edge interface cracks when $a/W = 0.1$. It was usually supposed that the stress intensity factors of a single edge interface crack are bigger than those of a double edge crack. However, as can be seen from Fig. 6 that for specific material combination, F_I and F_{II} of a double edge interface crack are also possibly bigger than that of a single edge crack.

4 Conlusions

The values of F_I, F_{II} in a bonded semi-infinite plate are completely obtained in this research. Moreover, Variations of F_I, F_{II} of the edge interface crack in a bonded plate subjected to tension and moment are also investigated with α, β and a/W. Meanwhile, stress intensity factors of a double edge interface crack are also discussed in this paper.

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