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STRESS CONCENTRATION OF A STRIP WITH A SINGLE EDGE NOTCH

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Abstract—This paper deals with the stress concentration analysis of a semicircular and a 60° V-shaped single edge notch in an infinite strip under three different types of loading conditions: (a) uniform tension, (b) in-plane bending and (c) pure tension. The stress field induced by a point force in the semi-infinite plate, Green's function in a closed form, is used to solve those problems. The results show that Neuber's formula gives an underestimated stress concentration factor when the notch is sharp and shallow. The stress concentration factors of 60° V-shaped notches are represented by diagrams for wide use.

1. INTRODUCTION

THE STRIP with the V-shaped notch shown in Fig. 1 is widely used as a test specimen of materials. However, there are few papers dealing with the single edge notch problem. Therefore, so-called Neuber's trigonometric formula, which gives approximate values of K_t[1], has been used for more than 40 years.

$$K_{tN} = \frac{(K_{ts}-1)(K_{td}-1)}{\sqrt{((K_{ts}-1)^2+(K_{td}-1)^2)}} + 1$$

where K_{ts} and K_{td} are stress concentration factors of shallow and deep notches respectively.

In this paper, the stress concentration problems of the 60° V-shaped single edge notch in the infinitely long strip under remote tension or in-plane bending are analysed by the body force method[2–4].

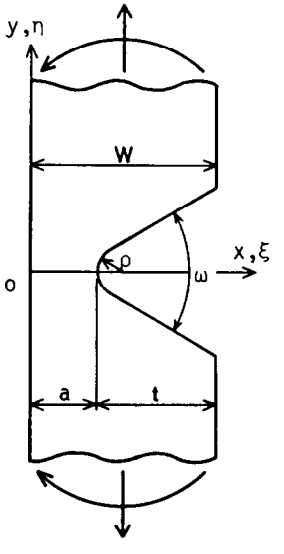


Fig. 1. A single edge notched strip under tension or in-plane bending.

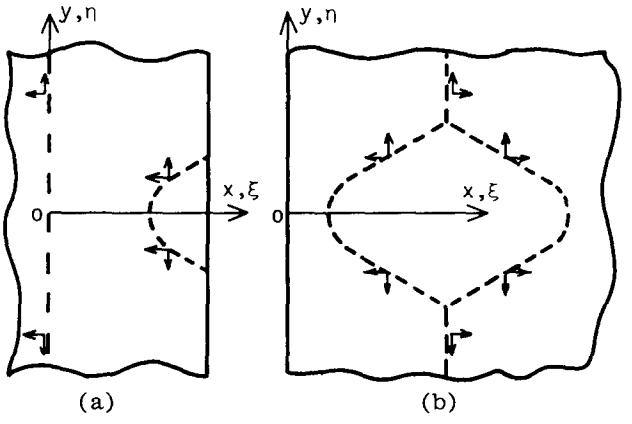


Fig. 2. Illustration of the present method of analysis.

2. METHOD OF ANALYSIS

2.1. *Fundamental solutions*

In the previous paper [5], we proposed the analysis method using Green's function of the semi-infinite plate to solve the problem of the strip having double edge notches. Since Green's function of the semi-infinite plate can be obtained in a closed form, it is convenient to use it in numerical analyses. In this paper, we are to apply the previous method to the problem of the single edge notch.

Figure 2 shows the present analysis method. Consider two kinds of semi-infinite plates: one is defined in the range $-\infty < x \leqslant W$ (Fig. 2a), the other in the range $0 \leqslant x < \infty$ (Fig. 2b). The edges $x = 0$ and $x = W$ correspond to the stress free edges of the strip shown in Fig. 1. Considering the effect of remaining stress at the edge upon the stress concentration factor in this analysis, we use Green's function of the semi-infinite plate shown in Fig. 2(a) for the analysis for shallow notches. On the other hand, for the analysis for deep notches, we use Green's function of the semi-infinite plate shown in Fig. 2(b). As a result, the method of analysis is reduced to determining the densities of body force (continuously embedded point forces) distributed in the semi-infinite plates shown in Fig. 2(a) or (b). The detailed procedure for numerical solutions and the equations of fundamental solutions will be omitted because they have been shown in the previous papers[4, 5]. In this paper we will focus on describing how to get the accurate solution for the problem of the single edge notch in the strip.

2.2 *Special remarks on the single edge notched strip*

Consider a strip having a single edge notch, which is subjected to tensile load P at infinity. From the equilibrium condition, the resultant P and the bending moment $M_p = Pt/2$ should act at the mid-point of the minimum section of the strip as shown in Fig. 3(a). Therefore, when we define the nominal stress for this problem, we should consider effects both of the resultant P and of the bending moment M_P. Finally, the stress concentration factor may be defined as follows.

$$\left.\begin{aligned} K_{ta} &= \frac{\sigma_{max}}{\sigma_{na}} \\ \sigma_{na} &= \frac{P}{a} + \frac{6M_P}{a^2} \\ &= \frac{P}{a}\left(1 + \frac{3t}{a}\right). \end{aligned}\right\} \tag{1}$$

On the other hand, when the bending moment M is applied at infinity of the strip with the single edge notch, only the bending moment M should act at the minimum section and the resultant should be zero. Thus, the stress concentration factor may be defined as follows.

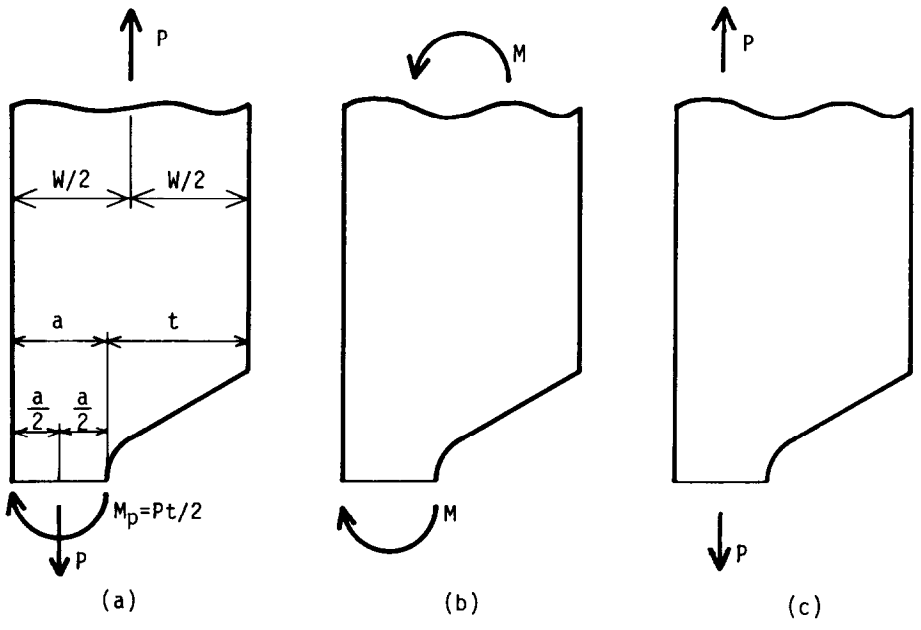


Fig. 3. Resultant and bending moment at the minimum section.

$$K_{tb} = \frac{\sigma_{max}}{\sigma_{nb}}, \qquad \sigma_{nb} = \frac{6M}{a^2}. \tag{2}$$

However, in the actual numerical analysis the body force densities that satisfy the boundary conditions completely for the notch and the strip edges cannot be obtained in the closed form. Thus, the values of the resultant and the bending moment at the minimum section would be different from the values mentioned above because the stress that should be zero at the boundaries would remain slightly. In the analysis for the single edge notch we have to exactly estimate both the resultant and the bending moment at the minimum section in order to get an accurate solution because this problem has no geometrical symmetry such as the double edge notched strip in the previous paper. Therefore, the following analysis method is used in this paper.

2.3 *Solution for the single edge notched strip*

First, a uniform tensile stress is applied as a boundary condition at infinity. Under this condition, the resultant P_1 at the minimum section, the bending moment M_1 and the maximum stress σ_{max1} are calculated numerically by the method described in the previous papers[4, 5]. Next, a bending stress is applied as a boundary condition at infinity. Under this condition, the resultant P_2, the bending moment M_2 and the maximum stress σ_{max2} are also calculated. Using both of those results, we can obtain the stress concentration factor K_{tc} for the case in which only resultant is produced at the minimum section and the bending moment equal to zero (Fig. 3c).

$$K_{tc} = \frac{\sigma_{max1} - \alpha_1 \sigma_{max2}}{P_1 - \alpha_1 P_2} a \tag{3}$$

where $\alpha_1 = M_1/M_2$. Moreover, we can estimate the stress concentration factor K_{tb} for the case in which only the bending moment is produced at the minimum section and the resultant equals to zero (Fig. 3b).

$$K_{tb} = \frac{\sigma_{max2} - \alpha_2 \sigma_{max1} a^2}{6(M_2 - \alpha_2 M_1)} \tag{4}$$

where $\alpha_2 = P_2/P_1$. Using eqs (3) and (4), we can finally obtain the stress concentration factor K_{ta}

for the case of uniform tension (Fig. 3a) by superposing the two loading conditions: (i) only the resultant P is produced at the minimum section (Fig. 3c); and (ii) only the bending moment $M_P = Pt/2$ is produced at the minimum section (Fig. 3b).

$$K_{ta} = \frac{K_{tc} + \alpha_3 K_{tb}}{1 + \alpha_3} \tag{5}$$

where $\alpha_3 = 3t/a$.

3. RESULTS AND DISCUSSION

3.1 *Stress concentration factors by Neuber's formula*

Neuber proposed an approximate formula (6) to estimate the stress concentration factors of the single edge notch in the strip under three different types of loading conditions[1].

$$K_{tN} = \frac{(K_{ts} - 1)(K_{td} - 1)}{\sqrt{((K_{ts} - 1)^2 + (K_{td} - 1)^2)}} + 1 \tag{6}$$

$$K_{ts} = 1 + 2\sqrt{\left(\frac{t}{\rho}\right)}. \tag{7}$$

(1) Pure tension (Fig. 3c)

$$K_{td} = \frac{\beta_1 - 2c}{1 - \dfrac{c}{\sqrt{(a/\rho + 1)}}} \tag{8}$$

(2) In-plane bending (Fig. 3b)

$$K_{td} = \frac{2(a/\rho + 1) - \beta_1\sqrt{(a/\rho + 1)}}{\dfrac{4}{\beta_2}\left(\dfrac{a}{\rho} + 1\right) - 3\beta_1}. \tag{9}$$

In eqs (8) and (9), β_1, β_2 and c are given as follows.

$$\beta_1 = \frac{2(a/\rho + 1)\sqrt{(a/\rho)}}{(a/\rho + 1)\tan^{-1}\sqrt{(a/\rho)} + \sqrt{(a/\rho)}}$$

$$\beta_2 = \frac{4(a/\rho)^{3/2}}{3\{\sqrt{(a/\rho)} + (a/\rho - 1)\tan^{-1}\sqrt{(a/\rho)}\}}$$

$$c = \frac{\beta_1 - \sqrt{(a/\rho + 1)}}{\dfrac{4}{3\beta_2}\sqrt{\left(\dfrac{a}{\rho} + 1\right)} - 1}$$

(3) Uniform tension at infinity (Fig. 3a)

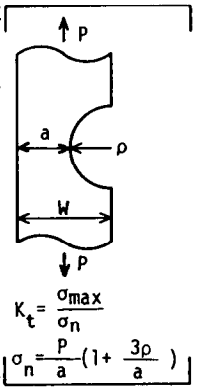
$$K_{td} = \frac{K'_{td} + \alpha_3 K''_{td}}{1 + \alpha_3} \tag{10}$$

where $\alpha_3 = 3t/a$. In eq. (10), K'_{td} and K''_{td} mean stress concentration factors given by eqs (8) and (9), respectively.

In the following section, we will compare the values given by Neuber's eq. (6) with the values obtained by the present analysis.

Table 1. SCFs of semicircular notches under uniform tension

ρ/W	Present analysis	Neuber
0.02	2.840	2.76
0.03	2.742	2.62
0.05	2.566	2.39
0.1	2.223	2.02
0.2	1.786	1.68
0.3	1.524	1.51
0.4	1.356	1.39
0.5	1.244	1.30
0.6	1.167	1.22
0.7	1.109	1.15
0.8	1.065	1.10
0.9	1.029	1.05



3.2 *Stress concentration factors of the semicircular notch*

In Tables 1–3, SCFs of the semicircular notch in the strip under uniform tension, in-plane bending, and pure tension are shown. The results in Tables 1, 2, and 3 are plotted in Figs 4, 5, and 6, respectively. In Figs 4–6, Neuber's corresponding values (eq. 6, $K_{ts} = 3.065$) are designated by the dashed line. Figures 7 and 8 show the stress distribution at the minimum section under three types of loading conditions.

Table 2. SCFs of semicircular notches under in-plane bending

ρ/W	Present analysis	Neuber
0.02	2.865	2.90
0.03	2.776	2.82
0.05	2.613	2.66
0.1	2.281	2.34
0.2	1.825	1.91
0.3	1.539	1.65
0.4	1.359	1.48
0.5	1.243	1.35
0.6	1.166	1.25
0.7	1.109	1.17
0.8	1.065	1.10
0.9	1.029	1.05

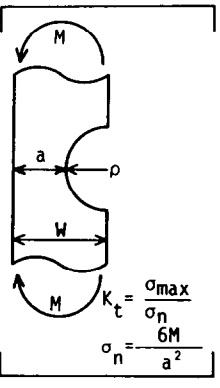
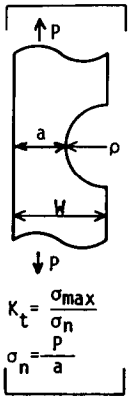


Table 3. SCFs of semicircular notches under pure tension

ρ/W	Present analysis	Neuber
0.02	2.838	2.75
0.03	2.738	2.60
0.05	2.559	2.34
0.1	2.204	1.91
0.2	1.756	1.51
0.3	1.504	1.32
0.4	1.352	1.22
0.5	1.249	1.15
0.6	1.172	1.10
0.7	1.112	1.07
0.8	1.066	1.04
0.9	1.030	1.02



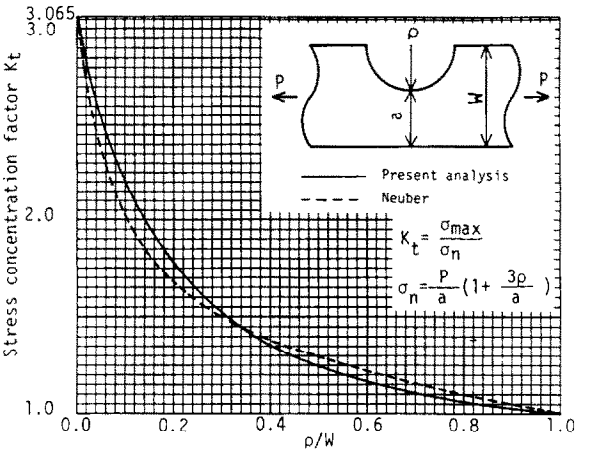


Fig. 4. SCF of semicircular notches under uniform tension.

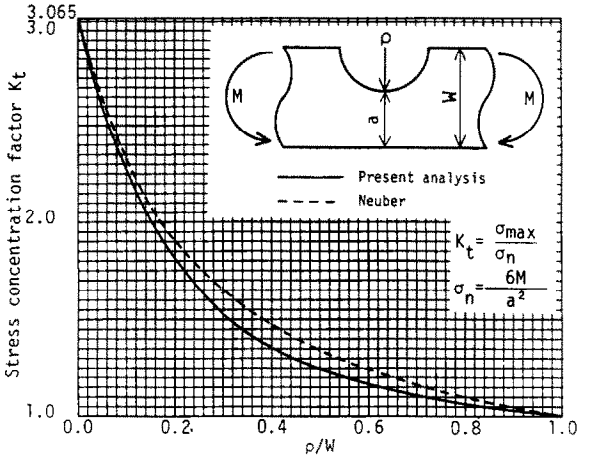


Fig. 5. SCF of semicircular notches under in-plane bending.

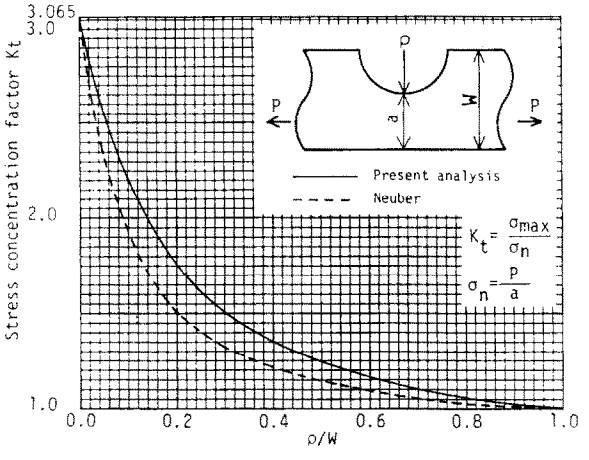
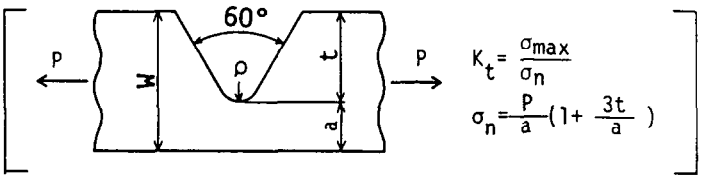


Fig. 6. SCF of semicircular notches under pure tension.

Table 4. SCFs of 60° V-shaped notches under uniform tension

	ρ/W=0.02		ρ/W=0.03		ρ/W=0.05		ρ/W=0.1		ρ/W=0.2		ρ/W=0.5		ρ/W=1.0	
t/W	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}
0.02	2.840	2.72	2.467	2.38	2.098	2.04	1.738	1.70	1.492	1.45	1.281	1.23	1.179	1.14
0.05	3.653	3.32	3.106	2.84	2.566	2.37	2.034	1.88	1.671	1.55	1.364	1.27	1.219	1.15
0.1	4.218	3.73	3.546	3.16	2.880	2.58	2.223	2.01	1.774	1.62	1.399	1.30	1.228	1.17
0.2	4.486	4.04	3.745	3.39	3.008	2.74	2.280	2.11	1.786	1.68	1.381	1.33	1.206	1.19
0.3	4.397	4.11	3.661	3.44	2.930	2.77	2.210	2.12	1.725	1.69	1.338	1.34	1.178	1.19
0.4	4.175	4.04	3.474	3.38	2.778	2.73	2.096	2.09	1.643	1.66	1.293	1.32	1.152	1.18
0.5	3.881	3.87	3.230	3.24	2.586	2.62	1.961	2.02	1.551	1.62	1.245	1.30	1.127	1.16
0.6	3.527	3.61	2.940	3.03	2.363	2.46	1.809	1.91	1.454	1.54	1.198	1.26	1.103	1.14
0.7	3.112	3.26	2.603	2.75	2.107	2.25	1.640	1.77	1.351	1.45	1.151	1.21	1.078	1.11
0.8	2.610	2.80	2.196	2.38	1.809	1.97	1.452	1.58	1.242	1.33	1.102	1.15	1.052	1.08
0.9	1.967	2.16	1.698	1.87	1.452	1.60	1.242	1.34	1.126	1.19	1.052	1.08	1.027	1.04



3.3 *Stress concentration factors of the 60° V-shaped notch*

Tables 4–6 show SCFs of the 60° V-shaped notch (K_t) under uniform tension, in-plane bending and pure tension. In the case of the shallow notch ($t \leq \rho/2$), K_t means SCF of partically-circular notch. Neuber's values K_{tN} (eq. 6) are also shown in order to be compared with the K_t values. The results in Tables 4–6 are plotted in Figs 9–14 so as to be useful further in design or research.

In Figs 9, 11 and 13, the ordinate represents the values of SCFs, and the abscissa represents the relative notch depth t/D. After comparing the values of the 60° V-shaped notch with corresponding Neuber's value, we conclude that Neuber's rule eq. (6) underestimates SCFs of the 60° V-shaped notch by about 12% for the worst case of uniform tension and by about 4% for the worst case of in-plane bending. Furthermore, it is found that Neuber's formula overestimates by about 12% for the worst case of in-plane bending. The charts of SCF are also shown in different ways from Figs 10, 12 and 14. In Figs 10, 12 and 14, the absissa represents the relative notch radius ρ/D. Using those charts (Figs 9–14), SCF K_t not calculated in this paper can be estimated.

Table 5. SCFs of 60° V-shaped notches under in-plane bending

	ρ/W=0.02		ρ/W=0.03		ρ/W=0.05		ρ/W=0.1		ρ/W=0.2		ρ/W=0.5		ρ/W=1.0	
t/W	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}
0.02	2.865	2.85	2.486	2.50	2.113	2.16	1.748	1.81	1.498	1.56	1.284	1.33	1.181	1.22
0.05	3.736	3.64	3.171	3.13	2.612	2.63	2.064	2.12	1.689	1.75	1.373	1.43	1.224	1.27
0.1	4.386	4.22	3.677	3.59	2.974	2.95	2.280	2.32	1.807	1.87	1.413	1.48	1.234	1.29
0.2	4.733	4.58	3.934	3.85	3.140	3.13	2.356	2.41	1.825	1.91	1.393	1.48	1.209	1.28
0.3	4.629	4.56	3.836	3.83	3.048	3.09	2.273	2.37	1.753	1.87	1.344	1.44	1.178	1.25
0.4	4.353	4.39	3.605	3.68	2.862	2.97	2.136	2.27	1.657	1.79	1.294	1.40	1.151	1.22
0.5	3.998	4.13	3.313	3.46	2.636	2.80	1.980	2.15	1.555	1.71	1.244	1.35	1.126	1.19
0.6	3.594	3.79	2.986	3.18	2.388	2.58	1.815	2.00	1.453	1.60	1.197	1.29	1.102	1.16
0.7	3.142	3.37	2.623	2.84	2.116	2.32	1.640	1.82	1.350	1.48	1.150	1.23	1.077	1.12
0.8	2.622	2.85	2.203	2.43	1.811	2.01	1.451	1.61	1.241	1.35	1.102	1.16	1.052	1.08
0.9	1.969	2.18	1.698	1.89	1.451	1.61	1.241	1.35	1.126	1.19	1.052	1.08	1.027	1.04

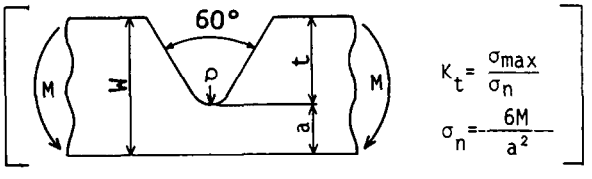
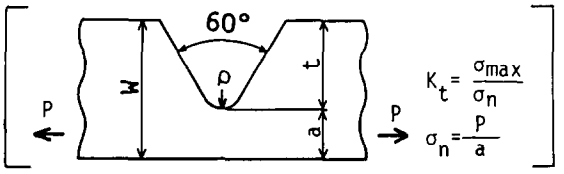
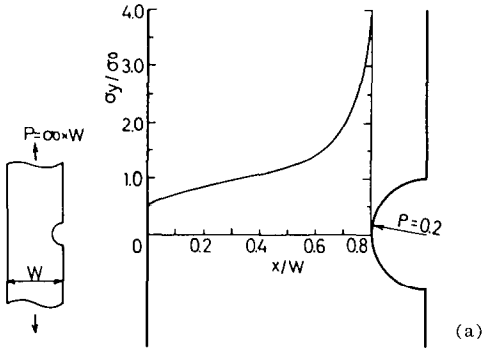
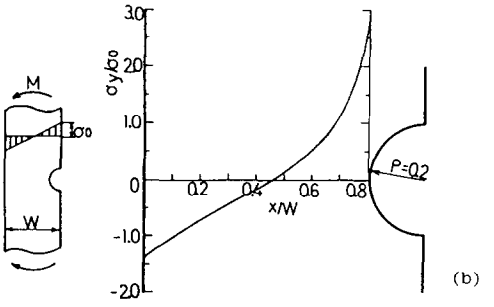


Table 6. SCFs of 60° V-shaped notches under pure tension

	ρ/W=0.02		ρ/W=0.03		ρ/W=0.05		ρ/W=0.1		ρ/W=0.2		ρ/W=0.5		ρ/W=1.0	
t/W	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}
0.02	2.839	2.71	2.466	2.38	2.098	2.04	1.738	1.69	1.491	1.44	1.280	1.23	1.179	1.13
0.05	3.640	3.26	3.096	2.80	2.558	2.32	2.029	1.85	1.668	1.52	1.363	1.25	1.219	1.14
0.1	4.162	3.57	3.502	3.01	2.849	2.46	2.204	1.91	1.763	1.54	1.395	1.25	1.226	1.13
0.2	4.301	3.64	3.603	3.05	2.909	2.46	2.223	1.88	1.756	1.51	1.372	1.23	1.203	1.12
0.3	4.097	3.52	3.435	2.94	2.777	2.36	2.129	1.81	1.689	1.46	1.331	1.20	1.178	1.11
0.4	3.819	3.33	3.211	2.78	2.609	2.24	2.016	1.72	1.615	1.40	1.291	1.18	1.154	1.09
0.5	3.529	3.09	2.980	2.58	2.436	2.09	1.902	1.63	1.540	1.34	1.249	1.15	1.130	1.08
0.6	3.226	2.81	2.737	2.35	2.253	1.92	1.779	1.52	1.458	1.28	1.204	1.12	1.104	1.06
0.7	2.883	2.48	2.461	2.10	2.044	1.73	1.636	1.41	1.363	1.22	1.155	1.09	1.079	1.05
0.8	2.468	2.10	2.119	1.80	1.789	1.52	1.464	1.28	1-253	1.15	1.104	1.06	1.053	1.03
0.9	1.926	1.63	1.691	1.45	1.464	1.28	1.253	1.15	1.130	1.08	1.053	1.03	1.026	1.02







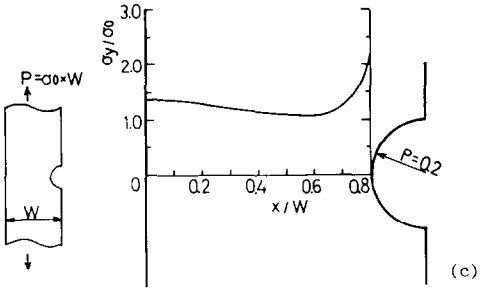


Fig. 7. Stress distribution at the minimum section ($\rho/D = 0.2$). (a) Uniform tension. (b) In-plane bending. (c) Pure tension.

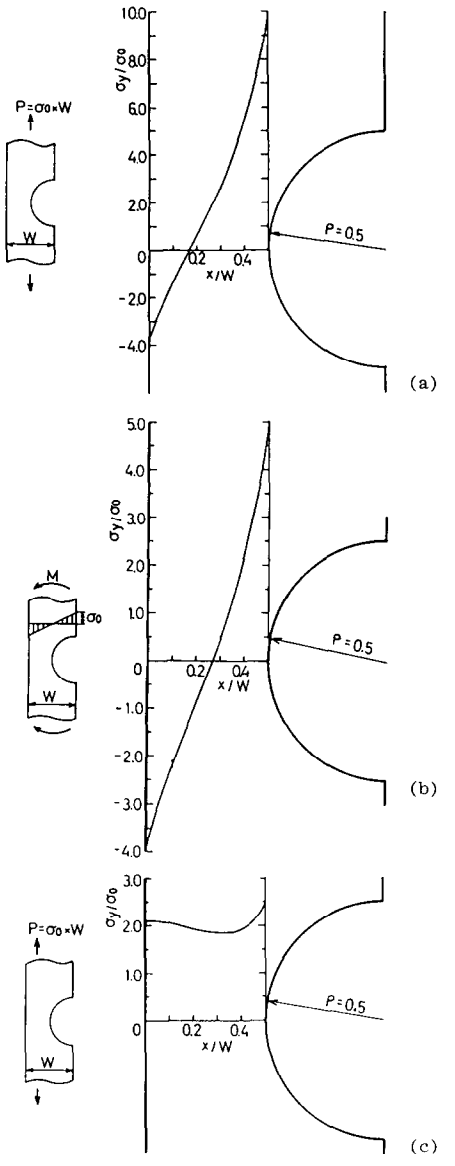


Fig. 8. Stress distribution at the minimum section ($\rho/D = 0.5$). (a) Uniform tension. (b) In-plane bending. (c) Pure tension.

3.4 *Special relationship between the notch shape and the stress concentration factor*

Tables 7–9 show the values K_t/K_{to}, where K_t means the SCF of double V-notches in the strip and K_{to} means the SCF of the notch in the semi-infinite plate [5]. In Tables 7–9, it is found that the values of K_t/K_{to} are mainly determined by the relative notch depth t/D alone, especially for shallow notches. Utilizing this fact, we can estimate the SCF of sharp V-notched strip not calculated in this paper ($\rho/D < 0.02$) from the SCF of the V-shaped semi-infinite plate [5].

4. CONCLUSION

In this paper, the stress concentration problems of the double V-notched strip under tension and in-plane bending were considered. Green's function of the semi-infinite plate in the closed form was used as a fundamental solution for strip problems. The conclusions can be made as follows:

(1) The stress concentration factors K_t of the 60° V-shaped notch were systematically calculated under various geometrical conditions. Neuber's formula is found to give an underestimated stress concentration factor by about 12% in tension case and by about 4% in

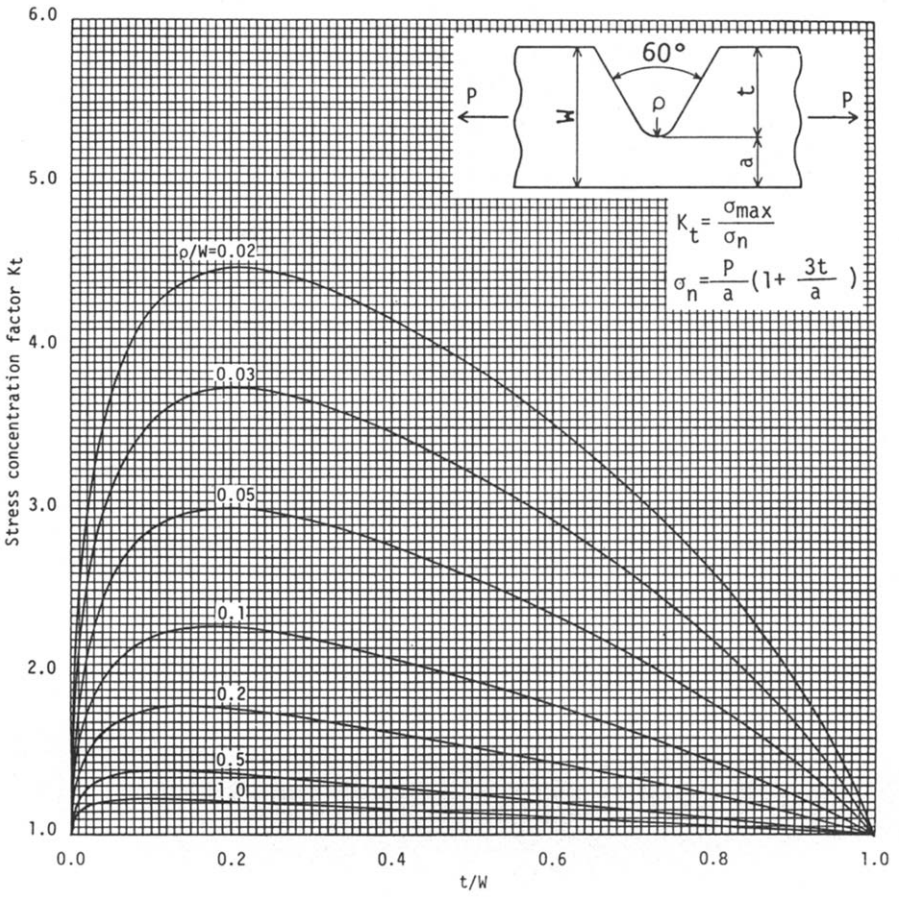


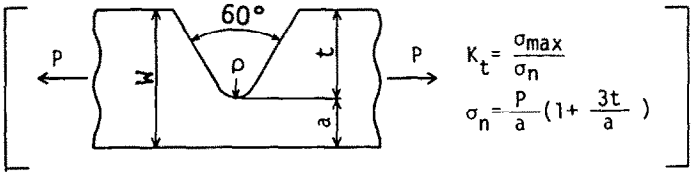
Fig. 9. SCF of 60° V-shaped notches under uniform tension.

bending case when the notch is sharp and shallow. On the other hand, the maximum error including overestimation is about 12% for both cases of tension and bending. The stress concentration factors were illustrated in the diagrams for design and research.

(2) The values K_t/K_{to} ($K_{to} = K_t|_{t/W\to 0}$) are found to be mainly determined by the relative notch depth t/W alone. Taking this into account, the K_t values of the extremely sharp notch in the strip not calculated in this paper can also be estimated from the values of K_{to}.

Table 7. Values $K_t/K_t|_{t/W\to 0}$ in uniform tension of a strip

ρ/W \ t/W	0.02	0.03	0.05	0.1	0.2	0.5	1.0
0.02	0.927	0.928	0.927	0.927	0.929	0.932	0.936
0.05	0.836	0.835	0.837	0.839	0.842	0.849	0.857
0.1	0.718	0.719	0.721	0.725	0.732	0.747	0.765
0.2	0.560	0.561	0.564	0.571	0.583	0.610	0.644
0.3	0.455	0.457	0.460	0.469	0.484	0.521	0.565
0.4	0.378	0.380	0.384	0.393	0.411	0.457	0.509
0.5	0.316	0.318	0.323	0.334	0.355	0.406	0.465
0.6	0.264	0.266	0.271	0.284	0.308	0.367	0.429
0.7	0.216	0.219	0.225	0.240	0.268	0.332	0.398
0.8	0.170	0.174	0.182	0.200	0.233	0.302	0.371
0.9	0.121	0.127	0.138	0.163	0.201	0.275	0.348



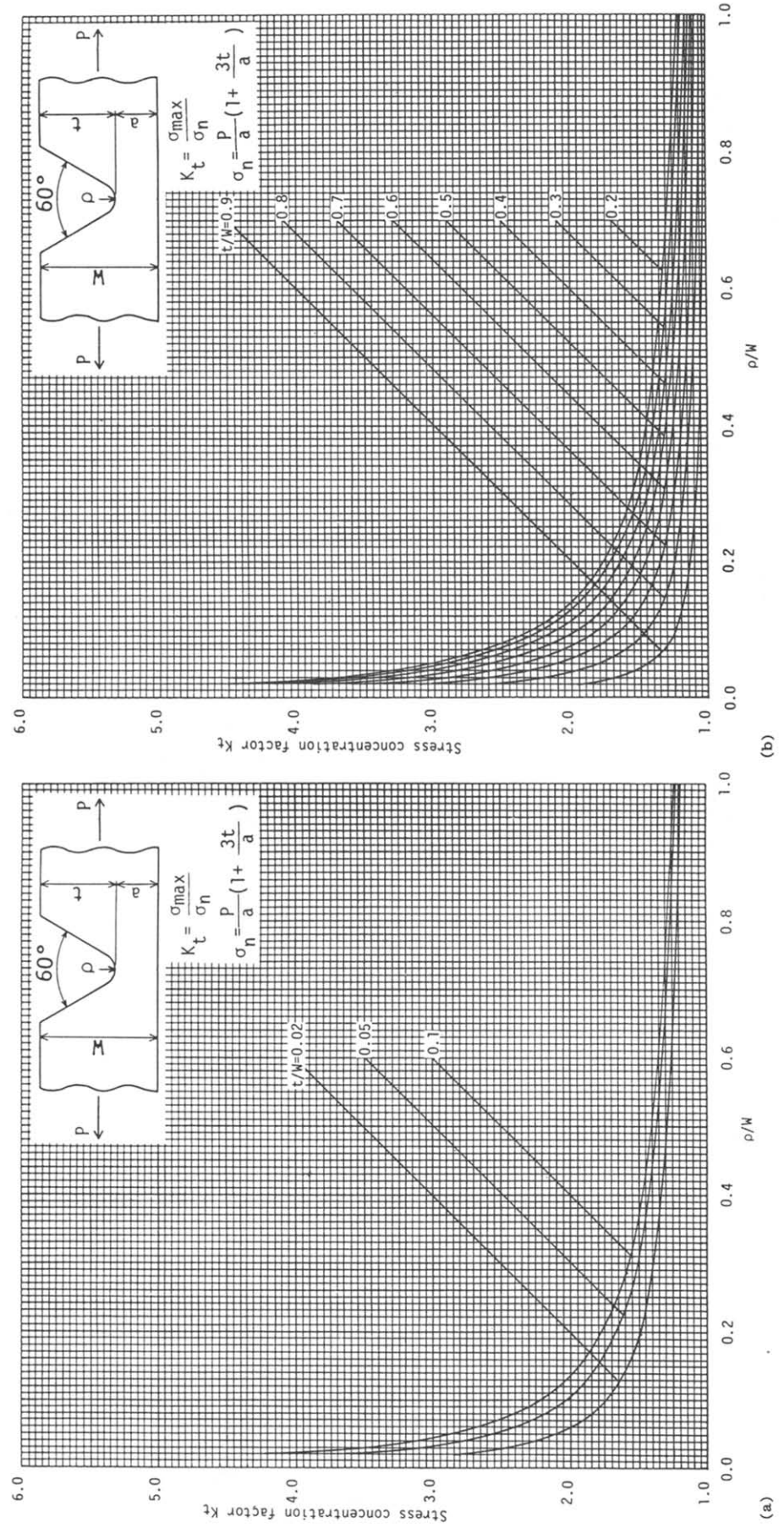


Fig. 10. (a), (b) SCF of 60° V-shaped notches under uniform tension.

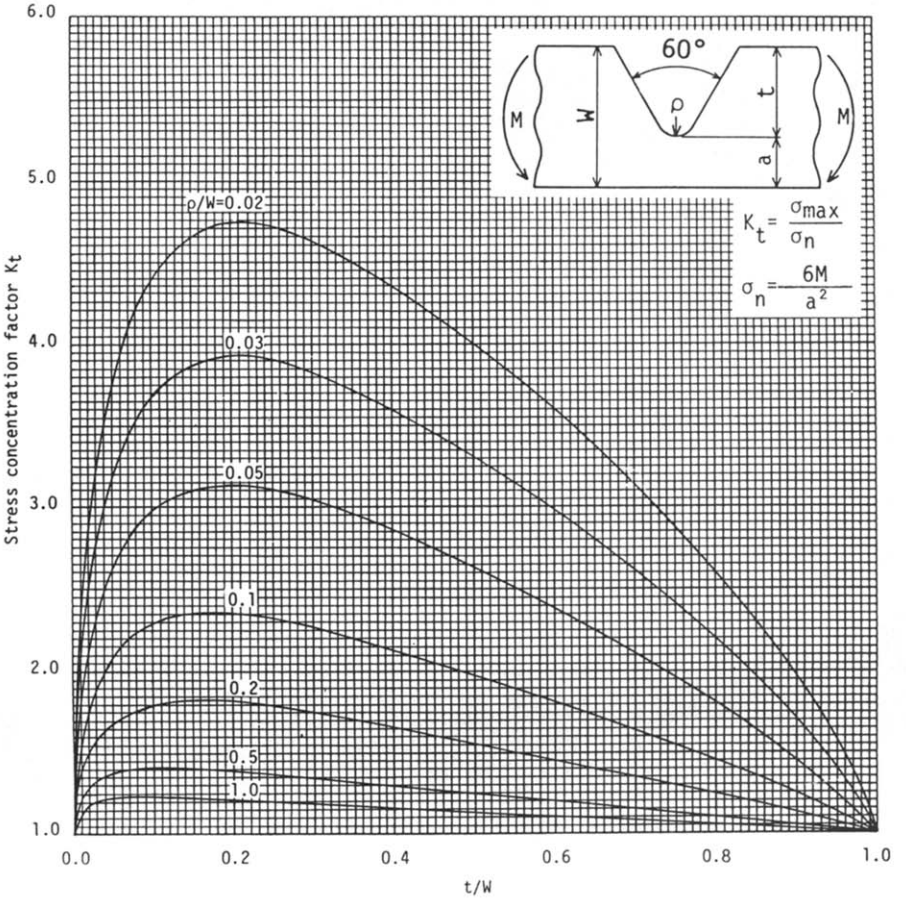
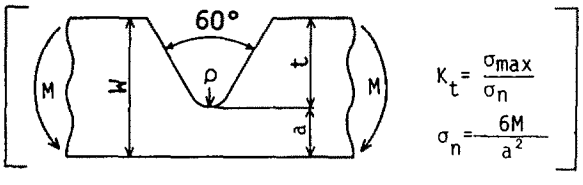


Fig. 11. SCF of 60° V-shaped notches under in-plane bending.

Table 8. Values $K_t/K_t|_{t/W\to 0}$ in in-plane bending of a strip

ρ/W / t/W	0.02	0.03	0.05	0.1	0.2	0.5	1.0
0.02	0.935	0.935	0.933	0.933	0.933	0.934	0.937
0.05	0.855	0.853	0.852	0.851	0.851	0.855	0.861
0.1	0.747	0.745	0.744	0.744	0.745	0.754	0.768
0.2	0.591	0.589	0.589	0.590	0.595	0.615	0.645
0.3	0.479	0.479	0.479	0.482	0.491	0.523	0.565
0.4	0.394	0.394	0.395	0.401	0.415	0.457	0.508
0.5	0.326	0.326	0.329	0.337	0.356	0.406	0.465
0.6	0.269	0.270	0.274	0.285	0.308	0.367	0.429
0.7	0.219	0.221	0.226	0.240	0.268	0.331	0.398
0.8	0.171	0.174	0.182	0.200	0.233	0.302	0.371
0.9	0.122	0.127	0.138	0.162	0.201	0.275	0.348



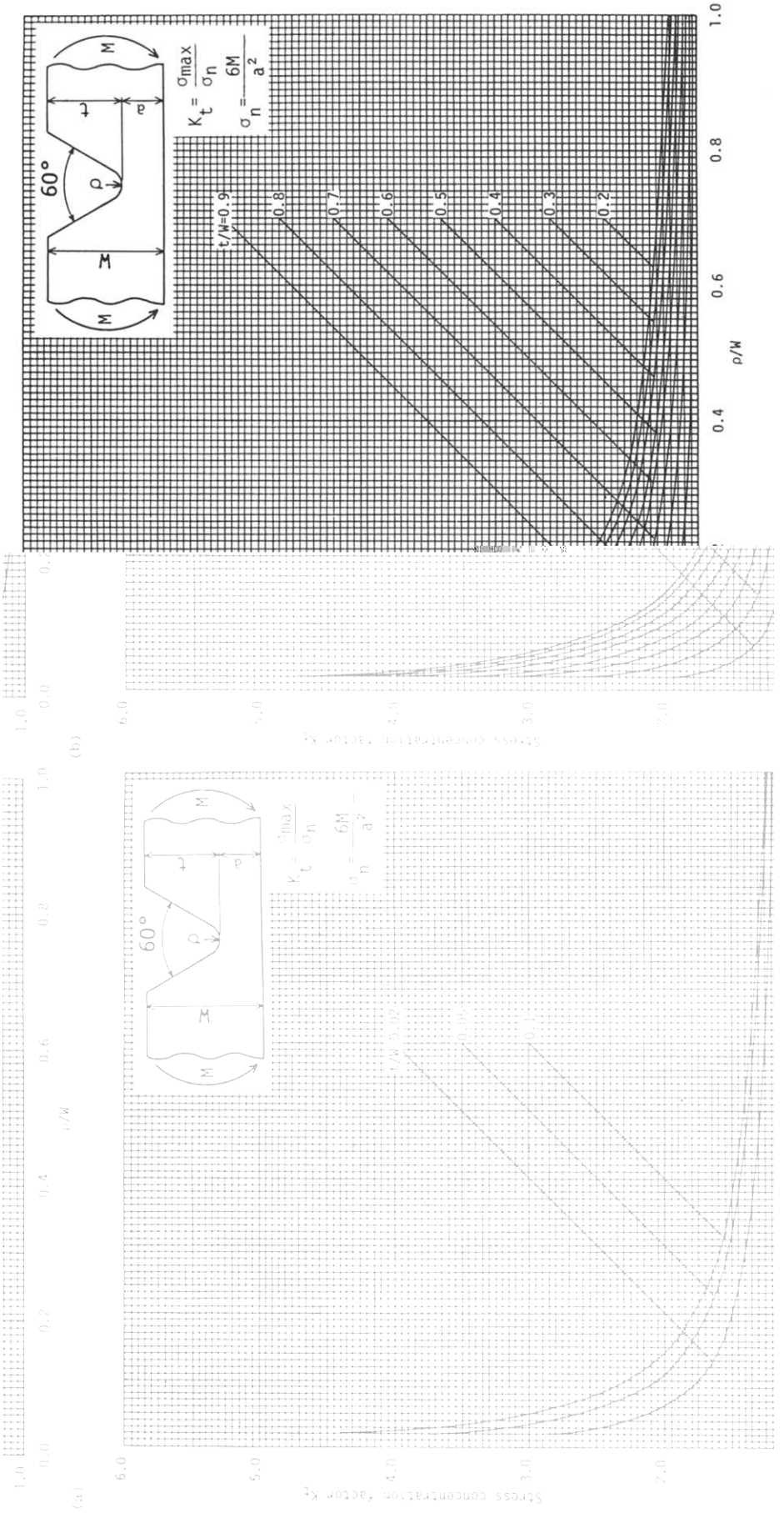


Fig. 12. (a), (b) SCF of 60° V-shaped notches under in-plane bending.