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# Stress concentration factors for round and flat test specimens with notches

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The stress concentration problem of round and flat bars with V-shaped notches under various loadings is especially important for test specimens used to investigate the fatigue strength of materials. Accurate stress concentration factors have been given in a recent analysis of the body force method. However, the results of the solutions have been presented in tabular form, which is not suitable for engineering applications. In this paper convenient formulae, which give the stress concentration factors with better than 1% accuracy, are proposed using the Neuber formula and the solution of a V-shaped notch in a semi-infinite plate. The stress concentration factors are also provided in a graphical way on the basis of the formulae.

**(Keywords: stress concentration factor; numerical analysis; notch; round test specimen; flat test specimen; tension; bending; torsion; in-plane bending; transverse bending)**

The stress concentration problem of round and flat bars with V-shaped notches, as shown in *Figure 1* is especially important for the test specimens used to investigate the fatigue strength of materials[1]. Approximate values of stress concentration factors given by the Neuber trigonometric rule[2], $K_{tN}$, have been used for more than 30 years. However, through systematic analyses using the body force method it has been confirmed that the Neuber values have non-conservative errors for a wide range of notch depth[3-15]. In these papers[3-15] the exact stress concentration factors have been illustrated in tables and charts; however, they have not been given in the form of formulae suitable for engineering applications. In this paper,

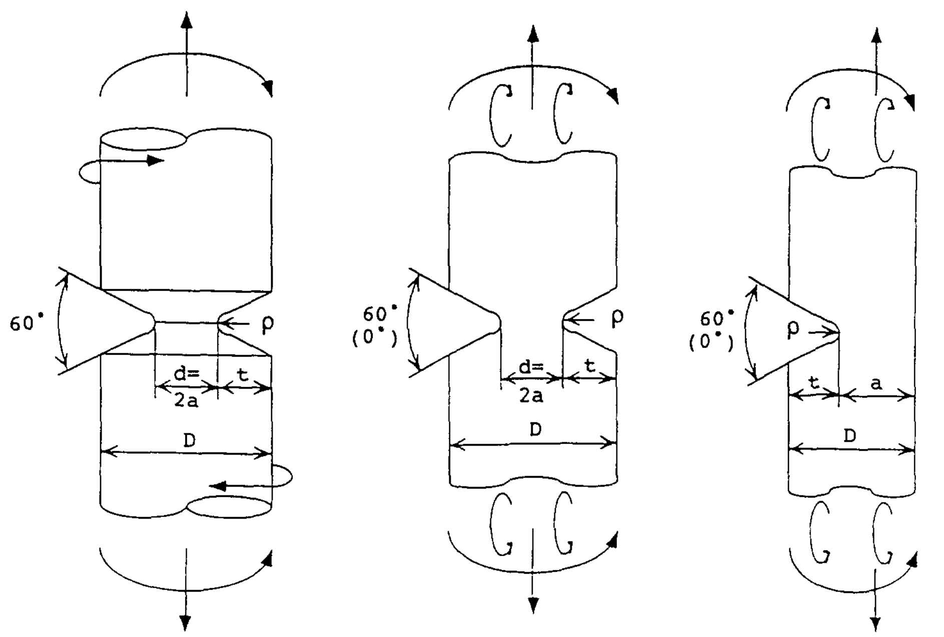


**Figure 1** Round and flat bars with V-shaped notches. Left: (a) tension; (b) bending; (c) torsion. Centre: (d) tension; (e) in-plane bending; (f) transverse bending (60° V-notch); (g) transverse bending (U-notch). Right: (h) pure tension; (i) in-plane bending; (j) transverse bending (60° V-notch); (k) transverse bending (U-notch)

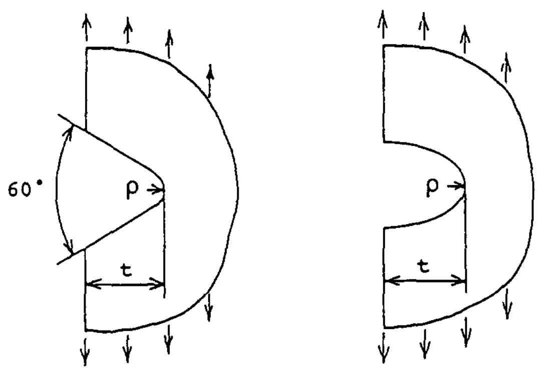


Figure 2 Semi-infinite plate with 60° V-shaped notch and semi-elliptical notch

therefore, convenient formulae will be proposed giving accurate stress concentration factors for a wide range of notch dimensions. In addition, the stress concentration factors are also provided in a graphical form on the basis of the formulae so that they can be used easily in design or research.

## DEFINITION OF STRESS CONCENTRATION FACTORS

In this paper, the stress concentration factors (SCFs) are based on the nominal stress at the minimum section and defined by

$$K_t = \frac{\sigma_{max}}{\sigma_n} \qquad (1)$$

where $\sigma_{max}$ is the maximum stress at the root of the notch. The problems treated in this paper are shown

Table 1 The values of $K_t/K_{tv}$ in problem (d)

| | $2\rho/D$ | | | | | | |
|---|---|---|---|---|---|---|---|
| $2t/D$ | 0.02 | 0.03 | 0.05 | 0.1 | 0.2 | 0.5 | 1.0 |
| 0.02 | 0.980 | 0.980 | 0.979 | 0.979 | 0.979 | 0.980 | 0.981 |
| 0.05 | 0.948 | 0.948 | 0.948 | 0.948 | 0.948 | 0.947 | 0.947 |
| 0.1 | 0.896 | 0.895 | 0.895 | 0.896 | 0.895 | 0.894 | 0.893 |
| 0.2 | 0.793 | 0.792 | 0.793 | 0.792 | 0.792 | 0.791 | 0.791 |
| 0.3 | 0.696 | 0.696 | 0.696 | 0.696 | 0.696 | 0.697 | 0.700 |
| 0.4 | 0.606 | 0.606 | 0.606 | 0.607 | 0.607 | 0.610 | 0.618 |
| 0.5 | 0.522 | 0.521 | 0.522 | 0.522 | 0.523 | 0.530 | 0.547 |
| 0.6 | 0.441 | 0.441 | 0.441 | 0.442 | 0.444 | 0.459 | 0.487 |
| 0.7 | 0.362 | 0.362 | 0.362 | 0.363 | 0.369 | 0.394 | 0.437 |
| 0.8 | 0.281 | 0.281 | 0.282 | 0.286 | 0.297 | 0.339 | 0.396 |
| 0.9 | 0.191 | 0.192 | 0.195 | 0.205 | 0.230 | 0.293 | 0.360 |

as follows with the definition of nominal net stress $\sigma_n$ for each problem (*Figure 1*):

(a) 60° V-shaped circumferential notched round bar under tension [$\sigma_n = 4P/(\pi d^2)$];
(b) 60° V-shaped circumferential notched round bar under bending [$\sigma_n = 32M/(\pi d^3)$];
(c) 60° V-shaped circumferential notched round bar under torsion [$\tau_n = 16T/(\pi d^3)$];
(d) 60° V-shaped double notched flat bar under tension [$\sigma_n = P/dh$];
(e) 60° V-shaped double notched flat bar under in-plane bending [$\sigma_n = 6M/d^2h$];
(f) 60° V-shaped double notched flat bar under transverse bending [$\sigma_n = 6M/dh^2$];
(g) U-shaped double notched flat bar under transverse bending [$\sigma_n = 6M/dh^2$];
(h) 60° V-shaped single notched flat bar under pure tension (no bending moment at the minimum section) [$\sigma_n = P/dh$];

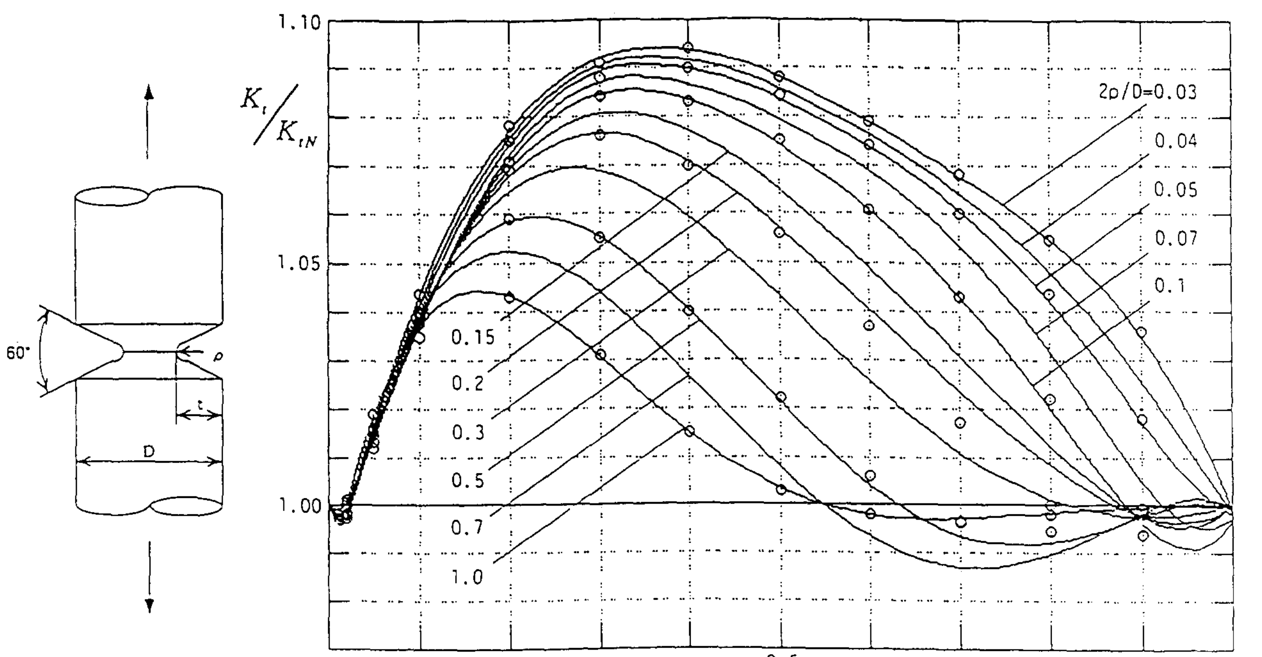


Figure 3 Curves of $K_t/K_{tN}$ given by the approximate formulae. $\sigma_n = 4P/(\pi d^2)$; $P$ = magnitude of external load; $d$ = diameter of minimum section

(i) 60° V-shaped single notched flat bar under in-plane bending [$\sigma_n = 6M/d^2h$];
(j) 60° V-shaped single notched flat bar under transverse bending [$\sigma_n = 6M/dh^2$];
(k) U-shaped single notched flat bar under transverse bending [$\sigma_n = 6M/dh^2$];

where $d$ is a diameter or width of minimum section, $h$ is a plate thickness, $P$ is the magnitude of the external load, $M$ is the magnitude of the external bending moment, and $T$ is the magnitude of the external tortional moment. Here, in most cases, the notch shape is assumed to be a 60° V-shape because the notch open angle has negligible effect on the SCFs in the range $0° \leq \theta \leq 90°$ except transverse bending problems.[15,16] In problems (a), (b), (f), (g), (j) and (k) Poisson's ratio $\nu$ is assumed to be 0.3. In this study the following notations will be used:

$$\xi = \sqrt{t/\rho}, \quad \eta = \sqrt{\rho/t}, \quad \lambda = 2t/D, \quad \epsilon = 2\rho/D \qquad (2)$$

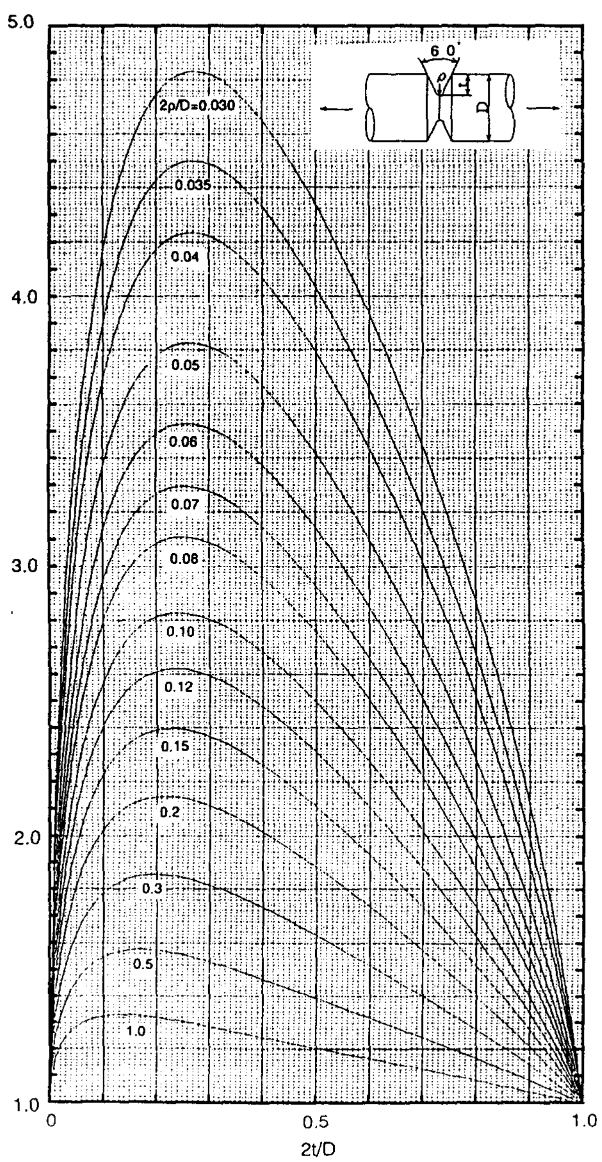


**Figure 4** $K_t$ of 60° V-shaped circumferential notched round bar under tension. $\sigma_n = 4P/(\pi d^2)$; $P$ = magnitude of external load; $d$ = diameter of minimum section

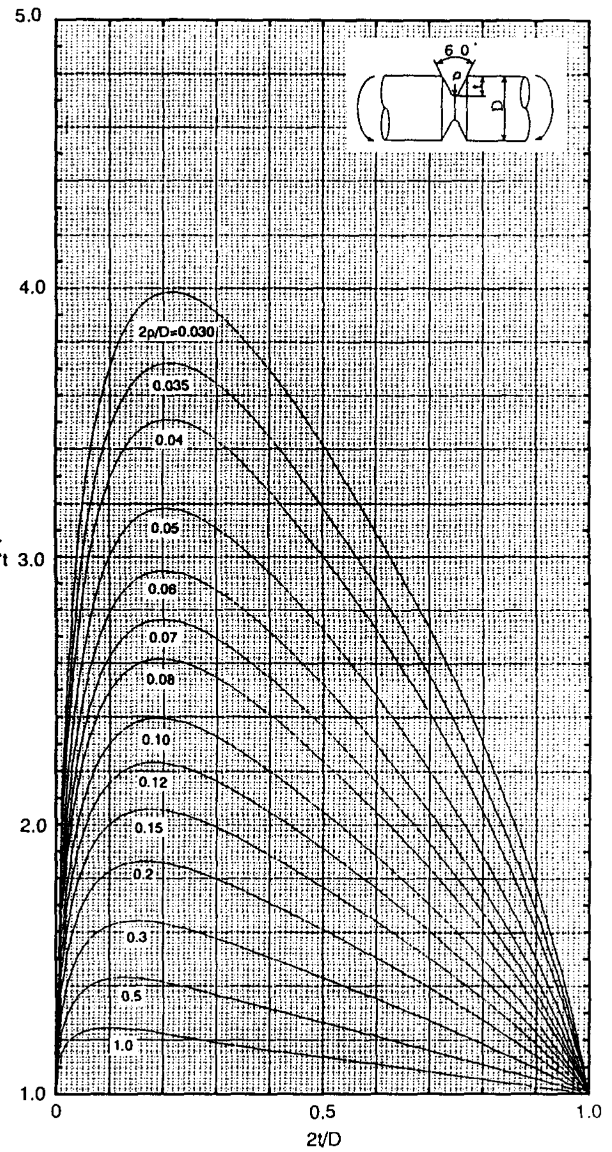


**Figure 5** $K_t$ of 60° V-shaped circumferential notched round bar under bending. $\sigma_n = 32M/(\pi d^3)$; $M$ = magnitude of external bending moment; $d$ = diameter of minimum section

where the parameters $\rho$, $t$, $D$, $d$ are indicated in *Figure 1*.

## APPROXIMATE FORMULAE FOR SHARP NOTCH USING THE SOLUTION OF THE SEMI-INFINITE PLATE

In general, it is difficult to calculate accurately the stress concentration factors for sharp notches ($\epsilon < 0.02$–$0.03$). However, for some cases the stress concentration factors can be estimated from the solution of a V-shaped notch in a semi-infinite plate, $K_{tv}$ (*Figure 2*). The reason is that the values of $K_t/K_{tv}$ ($K_{tv} = K_t|_{\lambda \to 0}$) are almost determined by $\lambda$ alone independent of notch shape for problems (a), (b), (d), (e), (h) and (i) unless the notch is very deep[5,7,8,11,13]. Here, $K_t$ corresponds to the notch in a round or flat bar and $K_{tv}$ corresponds to the notch in a semi-infinite plate, both having the same value of $t/\rho$. As an example, the values of $K_t/K_{tv}$ are shown in *Table 1* for problem (d).

First, the approximate formulae for $K_{tv}$ can be obtained through applying the least-square method to the results of the body force method[8,12]. They are shown in Equation (3) with less than 0.2% estimated errors. Here $K_{tH} = 1 + 2\sqrt{t/\rho}$.

$0 \leq \xi < 1.0$:

$$K_{tv} = (1.000 - 0.120\xi + 0.2683\xi^2 - 0.1273\xi^3)K_{tH} \tag{3a}$$

$0 < \eta \leq 1.0$ $(1.0 \leq \xi < \infty)$:

$$K_{tv} = (1.035 + 0.0261\eta - 0.1451\eta^2 + 0.0842\eta^3)K_{tE}$$
$$K_{tE} = (1.121 - 0.2846\eta + 0.3397\eta^2 - 0.1544\eta^3)K_{tH} \tag{3b}$$

Next, the $K_t$ values can be determined by $\lambda$ alone as shown in Equations (4)–(9) with the $K_{tv}$ values of the V-shaped notch in the semi-infinite plate with the same shape ratio $t/\rho$:

Problem (a):

$$K_t = (1.0 - 1.5183\lambda + 0.2530\lambda^2 + 2.2356\lambda^3 - 2.411\lambda^4)K_{tv} \tag{4}$$
$(\lambda \leq 0.5, \epsilon \leq 0.03)$

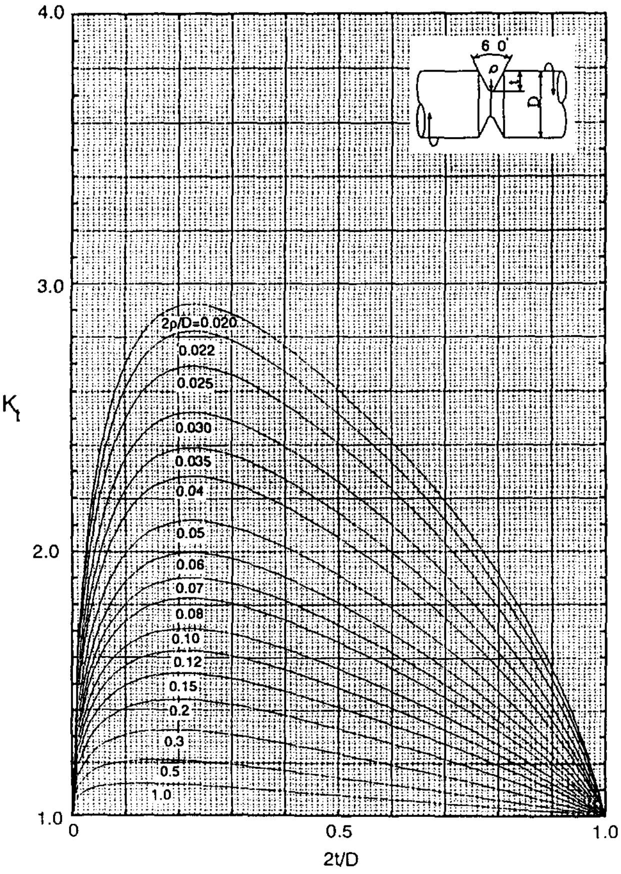


**Figure 6** $K_t$ of 60° V-shaped circumferential notched round bar under torsion. $\tau_n = 16T/(\pi d^3)$; $T$ = magnitude of external torsional moment; $d$ = diameter of minimum section

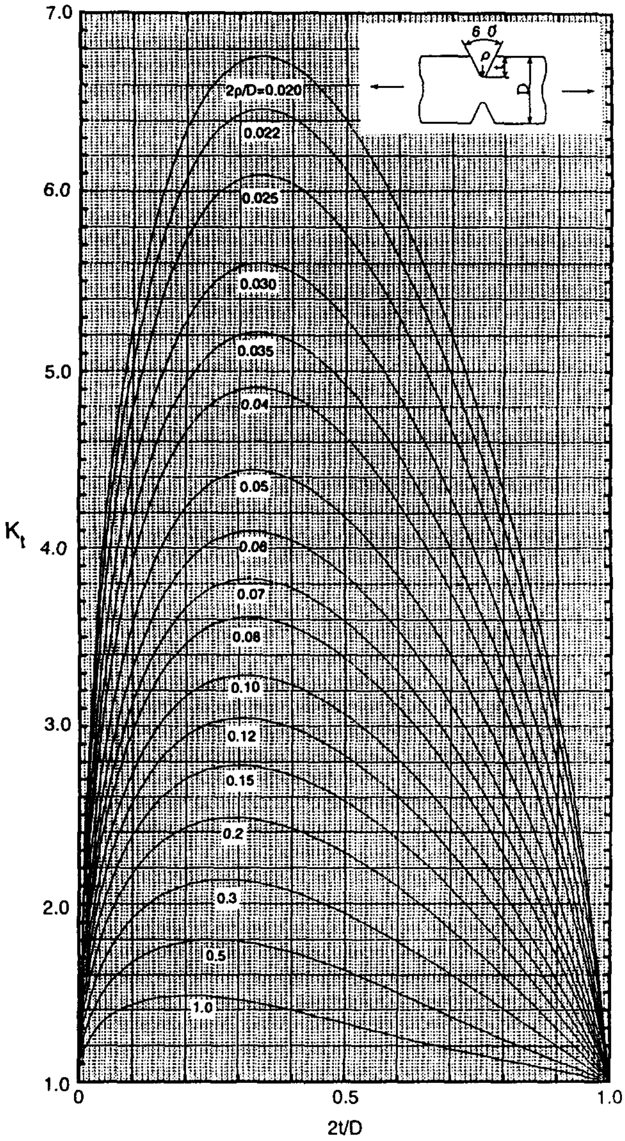


**Figure 7** $K_t$ of 60° V-shaped double notched flat bar under tension. $\sigma_n = P/dh$, $P$ = magnitude of external load; $d$ = width of minimum section; $h$ = plate thickness

Problem (b):

$$K_t = (1.0 - 3.0559\lambda + 1.5324\lambda^2 + 68.176\lambda^3 - 249.074\lambda^4)K_{tv} \tag{5}$$
$(\lambda \leq 0.2, \epsilon \leq 0.03)$

Problem (d):

$$K_t = (1.0 - 1.0340\lambda - 0.1447\lambda^2 + 0.9246\lambda^3 - 0.6667\lambda^4)K_{tv} \tag{6}$$
$(\lambda \leq 0.8, \epsilon \leq 0.02)$

Problem (e):

$$K_t = (1.0 - 2.7808\lambda + 9.7250\lambda^2 - 34.167\lambda^3)K_{tv}$$
$(\lambda \leq 0.1, \epsilon \leq 0.02)$ (7)

Problem (h):

$$K_t = (1.0 - 3.8913\lambda + 11.777\lambda^2 - 19.477\lambda^3)K_{tv}$$
$(\lambda \leq 0.1, \epsilon \leq 0.02)$ (8)

Problem (i):

$$K_t = (1.0 - 3.2698\lambda + 11.395\lambda^2 - 31.500\lambda^3)K_{tv}$$
$$(\lambda \leqslant 0.1, \epsilon \leqslant 0.02) \qquad (9)$$

## APPROXIMATE FORULAE USING THE SOLUTION OF THE NEUBER TRIGONOMETRIC RULE

Usually, the Neuber method makes use of the two exact solutions: that is, the solution of an elliptical hole in an infinite plate $K_{tH}$ as a shallow notch $K_{ts}$, and the solution of a hyperbolic notch as a deep notch, $K_{td}$. From these values, the Neuber value, $K_{tN}$, is given by the following ingenious simple equation[2]:

$$K_{tN} = \frac{(K_{ts} - 1)(K_{td} - 1)}{\sqrt{(K_{ts} - 1)^2 + (K_{td} - 1)^2}} + 1 \qquad (10)$$

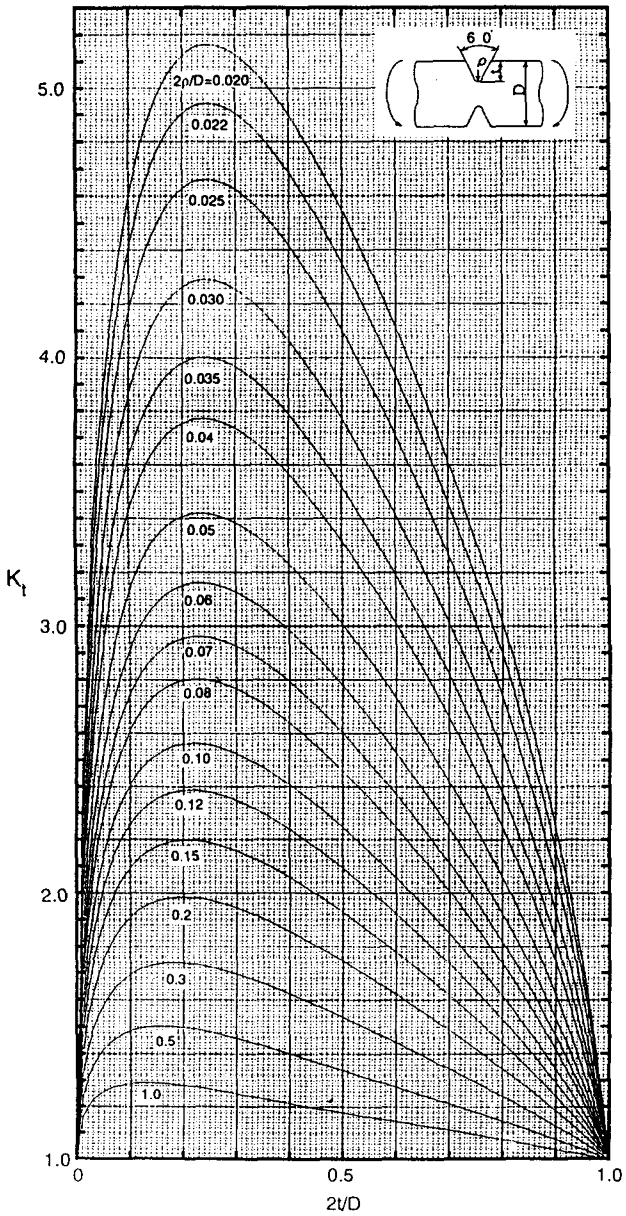


**Figure 8** $K_t$ of 60° V-shaped double notched flat bar under in-plane bending. $\sigma_n = 6M/dh^2$; $M$ = magnitude of external bending moment; $d$ = width of minimum section; $h$ = plate thickness

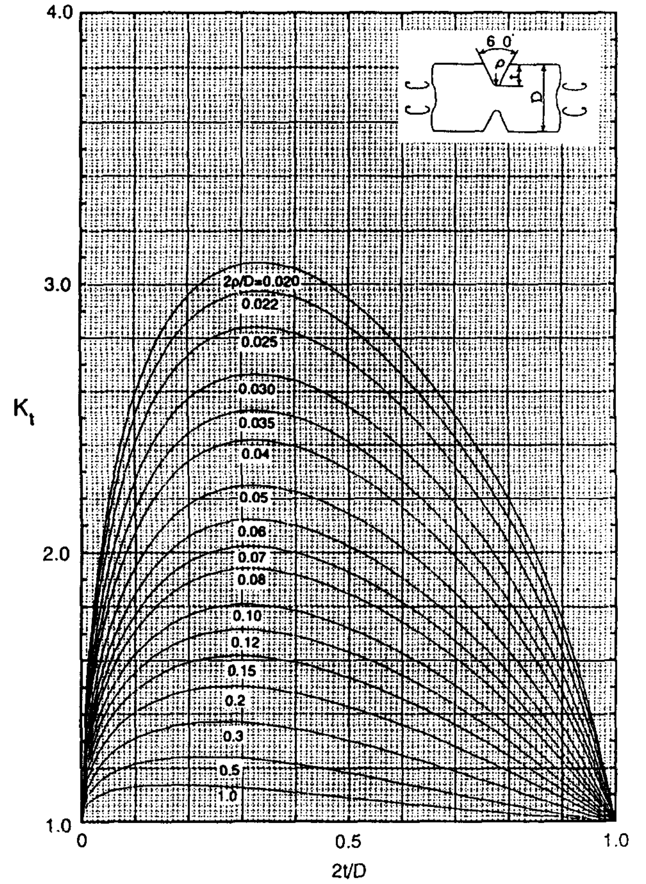


**Figure 9** $K_t$ of 60° V-shaped double notched flat bar under transverse bending. $\sigma_n = 6M/dh^2$; $M$ = magnitude of external bending moment; $d$ = width of minimum section; $h$ = plate thickness

In this paper, however, the more accurate solution (Equation (3)) of the 60° V-shaped notch in the semi-infinite plate will be used as a shallow notch, $K_{ts}$. Then, $K_{ts}$ and $K_{td}$ are expressed as follows.

Problem (a):

$$K_{ts} = K_{tv}$$

$$K_{td} = \frac{1}{N}\left\{\frac{a}{\rho}\sqrt{\left(\frac{a}{\rho}+1\right)} + (0.5+\nu)\frac{a}{\rho} + (1+\nu)\left[\sqrt{\left(\frac{a}{\rho}+1\right)}+1\right]\right\}$$

$$N = \frac{a}{\rho} + 2\nu\sqrt{\frac{a}{\rho}+1} + 2 \qquad (11)$$

Problem (b):

$$K_{ts} = K_{tv}$$

$$K_{td} = \frac{1}{N}\frac{3}{4}\left[\sqrt{\left(\frac{a}{\rho}+1\right)}+1\right]$$
$$\left[3\frac{a}{\rho} - (1-2\nu)\sqrt{\left(\frac{a}{\rho}+1\right)} + 4 + \nu\right]$$

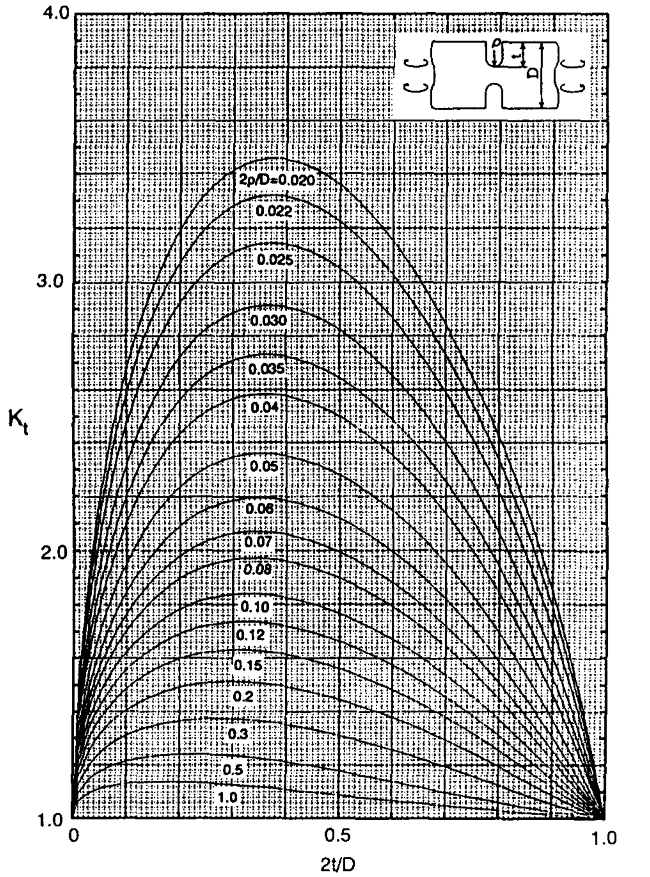

Figure 10 $K_t$ of U-shaped double notched flat bar under transverse bending. $\sigma_n = 6M/dh^2$; $M$ = magnitude of external bending moment; $d$ = width of minimum section; $h$ = plate thickness

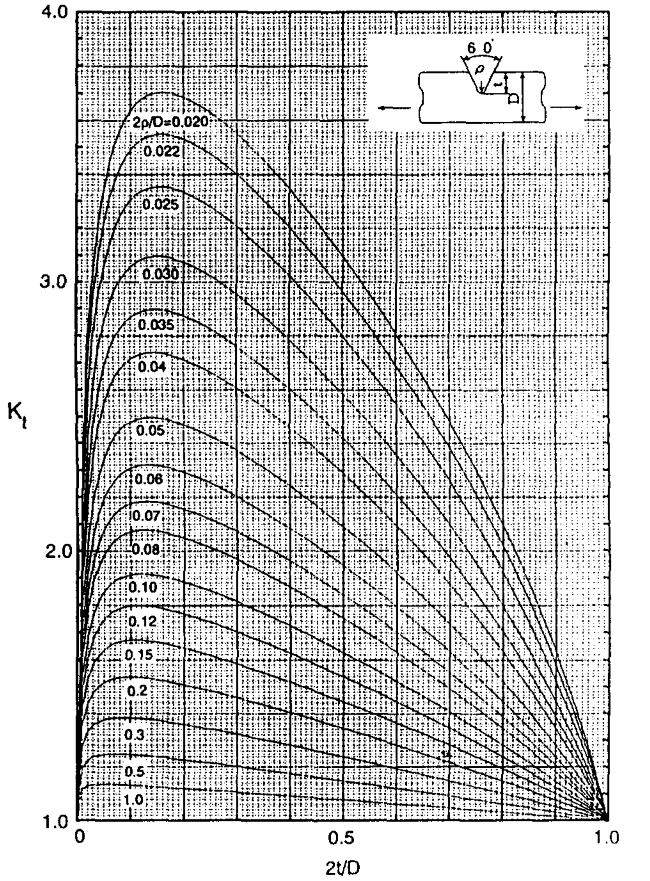

Figure 11 $K_t$ of 60° V-shaped single notched flat bar under pure tension (no bending moment at the minimum section). $\sigma_n = P/dh$; $P$ = magnitude of external load; $d$ = width of minimum section; $h$ = plate thickness

$$N = 3\left(\frac{a}{\rho}+1\right) + (1+4\nu)\sqrt{\left(\frac{a}{\rho}+1\right)} + (1+\nu)\Big/\left[1+\sqrt{\left(\frac{a}{\rho}+1\right)}\right] \tag{12}$$

Problem (c):

$$K_{ts} = 1 + \sqrt{\frac{t}{\rho}}$$

$$K_{td} = \frac{3(1+\sqrt{a/\rho+1})^2}{4(1+2\sqrt{a/\rho+1})} \tag{13}$$

Problem (d):

$$K_{ts} = K_{tv}$$

$$K_{td} = \frac{2(a/\rho+1)\sqrt{a/\rho}}{(a/\rho+1)\tan^{-1}\sqrt{a/\rho}+\sqrt{a/\rho}} \tag{14}$$

Problem (e):

$$K_{ts} = K_{tv}$$

$$K_{td} = \frac{4a/\rho \times \sqrt{a/\rho}}{3[\sqrt{a/\rho}+(a/\rho-1)\tan^{-1}\sqrt{a/\rho}]} \tag{15}$$

Problems (f) and (g):

$$K_{ts} = 1 + \frac{2(1+\nu)}{(3+\nu)}\sqrt{\frac{t}{\rho}}$$

$$K_{td} = \frac{2(1+\nu)\sqrt{a/\rho}}{(3+\nu)\tan^{-1}\sqrt{a/\rho}-(1-\nu)\sqrt{a/\rho}/(a/\rho+1)} \tag{16}$$

Problem (h):

$$K_{ts} = K_{tv}$$

$$K_{td} = \frac{\beta_1 - 2c}{1-\dfrac{c}{\sqrt{a/\rho+1}}} \tag{17}$$

Problem (i):

$$K_{ts} = K_{tv}$$

$$K_{td} = \frac{2(a/\rho+1) - \beta_1\sqrt{a/\rho+1}}{\dfrac{4}{\beta_2}\left(\dfrac{a}{\rho}+1\right) - 3\beta_1} \tag{18}$$

Problems (j) and (k):

$$K_{ts} = 1 + \frac{2(1+\nu)}{(3+\nu)}\sqrt{\frac{t}{\rho}}$$

$$K_{td} = \frac{2(1+\nu)\sqrt{a/\rho}}{(3+\nu)\tan^{-1}\sqrt{a/\rho} - (1-\nu)\sqrt{a/\rho}/(a/\rho+1)} \quad (19)$$

$$\beta_1 = \frac{2(a/\rho+1)\sqrt{a/\rho}}{(a/\rho+1)\tan^{-1}\sqrt{a/\rho} + \sqrt{a/\rho}}$$

$$\beta_2 = \frac{4(a/\rho)^{3/2}}{3\{\sqrt{a/\rho} + (a/\rho-1)\tan^{-1}\sqrt{a/\rho}\}}$$

$$c = \frac{\beta_1 - \sqrt{a/\rho+1}}{\frac{4}{3\beta_2}\sqrt{\frac{a}{\rho}+1} - 1}$$

In this paper, approximate formulae with high accuracy are proposed by applying the least-square method to the ratio $K_t/K_{tN}$, where exact $K_t$ values can be found in the previous papers[3–15]. These obtained formulae are shown in the Appendix. As an example, the curves given by the formula for problem (a) is shown in *Figure 3*. The symbol ○ in the diagram denotes the exact result of the body force method. *Figure 3* indicates that the proposed approximate formulae give the stress concentration factors to better than 1% accuracy.

The stress concentration factors for problems (a)–(k) are provided directly in a graphical way as shown in *Figures 4–14* so that they can be used easily in design or research. *Tables 2–12* also show the stress concentration factors for test specimens as a standard type obtained through two types of approximate formula (Equations (3)–(30)). The specimen geometries shown in *Tables 2–12* have been used by Nisitani *et al.*[17,18] in fatigue experiments.

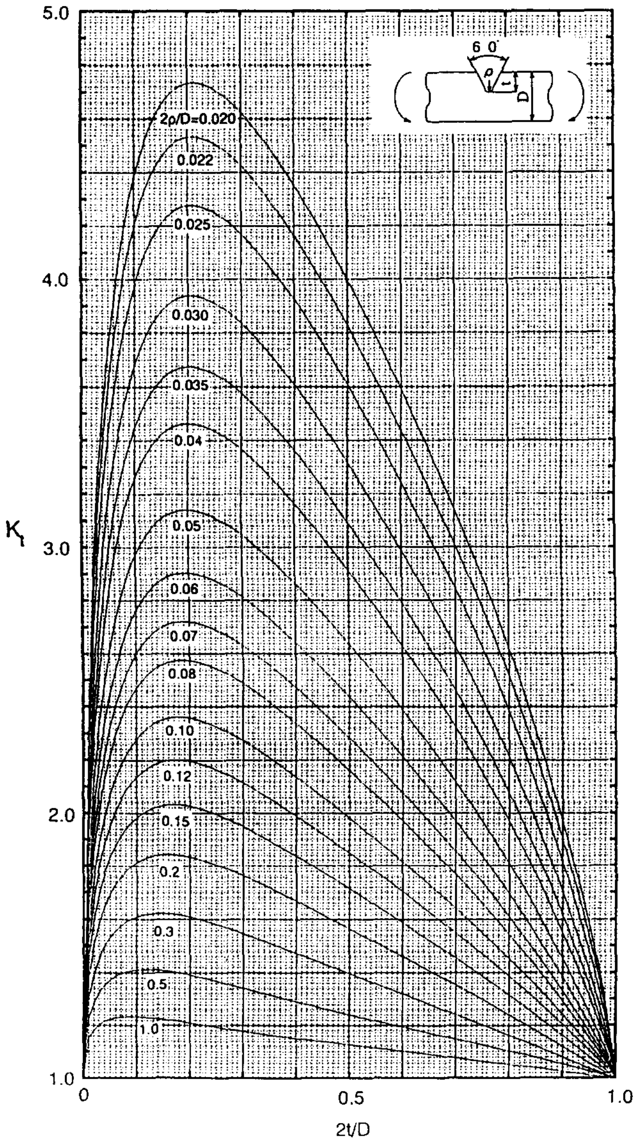


**Figure 12** $K_t$ of 60° V-shaped single notched flat bar under in-plane bending. $\sigma_n = 6M/d^2h$; $M$ = magnitude of external bending moment; $d$ = width of minimum section; $h$ = plate thickness

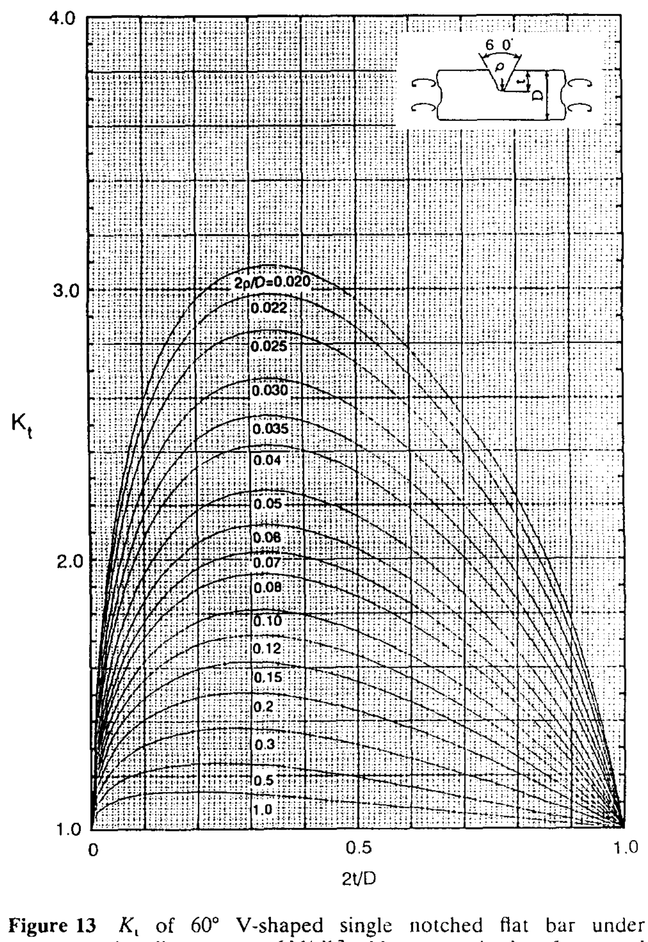


**Figure 13** $K_t$ of 60° V-shaped single notched flat bar under transverse bending. $\sigma_n = 6M/dh^2$; $M$ = magnitude of external bending moment; $d$ = width of minimum section; $h$ = plate thickness

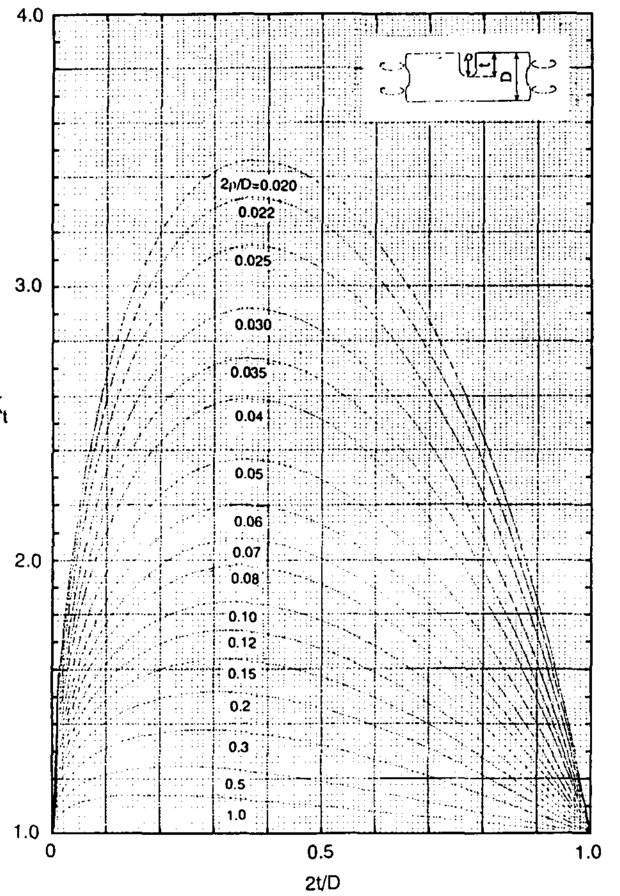


**Figure 14** $K_t$ of U-shaped single notched flat bar under transverse bending. $\sigma_n = 6M/dh^2$; $M$ = magnitude of external bending moment; $d$ = width of minimum section; $h$ = plate thickness

## CONCLUSIONS

The stress concentration of round and flat bars with V-shaped notches under various loading is especially important for the test specimens used to investigate the fatigue strength of materials. Accurate stress concentration factors have been given in a recent analysis of the body force method. In this paper, approximate formulae that are suitable for engineering applications are proposed. The conclusions can be made as follows.

1. Approximate formulae with high accuracy are proposed by applying the least-square method to the ratio $K_t/K_{tN}$, where $K_t$ is the result obtained by the body force method and $K_{tN}$ is the result of the Neuber trigonometric rule. In the range $0.02 \leq 2\rho/D \leq 1.0$, the formulae give stress concentration factors of better than 1% accuracy.
2. Numerical values have not been obtained because of the difficulty in the numerical analysis for sharp notches. However, in some cases, approximate formulae can be obtained from the solution of a V-shaped notch in a semi-infinite plate. For sharp notches ($2\rho/D<0.03$), the approximate formulae give the stress concentration factors with better than 1% accuracy unless the notch is very deep.
3. The stress concentration factors are provided in a graphical way on the basis of the proposed formulae so that they can be used easily in design and research. The SCFs obtained through two types of formula are also shown in tables as a standard type for test specimens in fatigue experiments.

**Table 2** $K_t$ of 60° V-shaped circumferential notched round bar under tension

| | $t/d$ = 0.01 | | $t/d$ = 0.02 | | $t/d$ = 0.03 | | $t/d$ = 0.04 | | $t/d$ = 0.05 | | $t/d$ = 0.06 | | $t/d$ = 0.10 | | $t/d$ = 0.20 | | $t/d$ = 0.50 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho/d$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ |
| 0.001 | 7.771 | 7.665 | 10.396 | 10.097 | 12.229 | 11.722 | 13.635 | 12.929 | 14.762 | 13.874 | 15.689 | 14.640 | 18.181 | 16.675 | 20.839 | 18.951 | 22.735 | 20.879 |
| 0.002 | 5.704 | 5.639 | 7.545 | 7.349 | 8.831 | 8.492 | 9.818 | 9.339 | 10.609 | 10.002 | 11.259 | 10.538 | 13.003 | 11.958 | 14.854 | 13.536 | 16.158 | 14.860 |
| 0.005 | 3.886 | 3.856 | 5.029 | 4.922 | 5.828 | 5.633 | 6.441 | 6.160 | 6.932 | 6.570 | 7.335 | 6.901 | 8.414 | 7.772 | 9.546 | 8.727 | 10.322 | 9.515 |
| 0.010 | 2.975 | 2.961 | 3.773 | 3.707 | 4.326 | 4.200 | 4.750 | 4.562 | 5.088 | 4.844 | 5.366 | 5.070 | 6.106 | 5.662 | 6.874 | 6.301 | 7.382 | 6.818 |
| 0.020 | 2.352 | 2.348 | 2.880 | 2.851 | 3.256 | 3.191 | 3.540 | 3.438 | 3.765 | 3.627 | 3.951 | 3.779 | 4.453 | 4.170 | 4.989 | 4.584 | 5.305 | 4.910 |
| 0.030 | 2.082 | 2.080 | 2.505 | 2.481 | 2.795 | 2.744 | 3.020 | 2.941 | 3.198 | 3.090 | 3.343 | 3.208 | 3.737 | 3.509 | 4.163 | 3.823 | 4.424 | 4.066 |
| 0.040 | 1.923 | 1.923 | 2.281 | 2.261 | 2.526 | 2.483 | 2.711 | 2.643 | 2.861 | 2.769 | 2.983 | 2.868 | 3.312 | 3.116 | 3.664 | 3.370 | 3.869 | 3.564 |
| 0.050 | 1.817 | 1.818 | 2.129 | 2.112 | 2.344 | 2.305 | 2.506 | 2.444 | 2.633 | 2.550 | 2.740 | 2.635 | 3.025 | 2.848 | 3.325 | 3.062 | 3.491 | 3.223 |
| 0.060 | 1.740 | 1.741 | 2.018 | 2.002 | 2.211 | 2.174 | 2.356 | 2.297 | 2.470 | 2.390 | 2.562 | 2.464 | 2.814 | 2.650 | 3.075 | 2.835 | 3.214 | 2.972 |
| 0.080 | 1.632 | 1.633 | 1.865 | 1.850 | 2.026 | 1.991 | 2.146 | 2.092 | 2.240 | 2.167 | 2.316 | 2.226 | 2.519 | 2.374 | 2.726 | 2.518 | 2.826 | 2.623 |
| 0.100 | 1.560 | 1.560 | 1.762 | 1.747 | 1.900 | 1.867 | 2.003 | 1.952 | 2.084 | 2.015 | 2.149 | 2.065 | 2.318 | 2.186 | 2.490 | 2.304 | 2.565 | 2.388 |
| 0.200 | 1.383 | 1.381 | 1.508 | 1.495 | 1.593 | 1.563 | 1.655 | 1.609 | 1.702 | 1.643 | 1.739 | 1.669 | 1.832 | 1.731 | 1.913 | 1.787 | 1.930 | 1.826 |
| 0.300 | 1.306 | 1.301 | 1.397 | 1.383 | 1.460 | 1.430 | 1.504 | 1.461 | 1.537 | 1.483 | 1.562 | 1.500 | 1.623 | 1.538 | 1.668 | 1.572 | 1.663 | 1.595 |
| 0.400 | 1.261 | 1.253 | 1.332 | 1.317 | 1.381 | 1.352 | 1.416 | 1.375 | 1.441 | 1.390 | 1.460 | 1.402 | 1.503 | 1.428 | 1.528 | 1.451 | 1.512 | 1.466 |
| 0.500 | 1.230 | 1.221 | 1.289 | 1.272 | 1.329 | 1.300 | 1.356 | 1.317 | 1.376 | 1.329 | 1.391 | 1.337 | 1.423 | 1.357 | 1.436 | 1.373 | 1.413 | 1.384 |
| 1.000 | | 1.139 | | 1.164 | | 1.175 | | 1.182 | | 1.186 | | 1.189 | | 1.196 | | 1.201 | 1.209 | 1.205 |
| 2.000 | | 1.083 | | 1.093 | | 1.097 | | 1.099 | | 1.101 | | 1.102 | | 1.104 | | 1.105 | | 1.106 |
| 10.000 | | 1.021 | | 1.021 | | 1.021 | | 1.022 | | 1.022 | | 1.022 | | 1.022 | | 1.022 | | 1.022 |
| 20.000 | | 1.011 | | 1.011 | | 1.011 | | 1.011 | | 1.011 | | 1.011 | | 1.011 | | 1.011 | | 1.011 |

$\sigma_n = 4P/(\pi d^2)$; $P$ = magnitude of external load; $d$ = diameter of minimum section

**Table 3** $K_t$ of 60° V-shaped circumferential notched round bar under bending

| | $t/d = 0.01$ | | $t/d = 0.02$ | | $t/d = 0.03$ | | $t/d = 0.04$ | | $t/d = 0.05$ | | $t/d = 0.06$ | | $t/d = 0.10$ | | $t/d = 0.20$ | | $t/d = 0.50$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho/d$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ |
| 0.001 | 7.537 | 7.413 | 9.799 | 9.485 | 11.247 | 10.758 | 12.294 | 11.641 | 13.116 | 12.295 | 13.800 | 12.801 | 15.659 | 14.047 | | 15.274 | | 16.188 |
| 0.002 | 5.532 | 5.459 | 7.112 | 6.909 | 8.122 | 7.797 | 8.853 | 8.411 | 9.426 | 8.864 | 9.903 | 9.214 | 11.199 | 10.069 | | 10.905 | | 11.520 |
| 0.005 | 3.769 | 3.738 | 4.741 | 4.634 | 5.360 | 5.177 | 5.808 | 5.551 | 6.159 | 5.825 | 6.452 | 6.035 | 7.246 | 6.544 | | 7.033 | | 7.387 |
| 0.010 | 2.885 | 2.876 | 3.557 | 3.495 | 3.979 | 3.864 | 4.282 | 4.115 | 4.521 | 4.298 | 4.720 | 4.438 | 5.259 | 4.772 | | 5.088 | | 5.311 |
| 0.020 | 2.282 | 2.283 | 2.723 | 2.692 | 3.012 | 2.941 | 3.214 | 3.106 | 3.366 | 3.224 | 3.484 | 3.314 | 3.772 | 3.525 | | 3.720 | | 3.855 |
| 0.030 | 2.019 | 2.024 | 2.362 | 2.345 | 2.582 | 2.533 | 2.740 | 2.661 | 2.857 | 2.751 | 2.948 | 2.819 | 3.167 | 2.977 | 3.351 | 3.120 | 3.417 | 3.217 |
| 0.040 | 1.866 | 1.872 | 2.149 | 2.138 | 2.333 | 2.294 | 2.460 | 2.396 | 2.557 | 2.471 | 2.631 | 2.526 | 2.809 | 2.652 | 2.955 | 2.765 | 3.004 | 2.841 |
| 0.050 | 1.763 | 1.769 | 2.008 | 1.998 | 2.165 | 2.131 | 2.274 | 2.218 | 2.355 | 2.279 | 2.418 | 2.326 | 2.567 | 2.432 | 2.687 | 2.525 | 2.724 | 2.587 |
| 0.060 | 1.687 | 1.694 | 1.903 | 1.896 | 2.042 | 2.012 | 2.138 | 2.087 | 2.208 | 2.140 | 2.261 | 2.180 | 2.392 | 2.271 | 2.491 | 2.350 | 2.520 | 2.402 |
| 0.080 | 1.583 | 1.590 | 1.759 | 1.753 | 1.872 | 1.845 | 1.949 | 1.905 | 2.006 | 1.946 | 2.049 | 1.977 | 2.148 | 2.047 | 2.220 | 2.107 | 2.237 | 2.145 |
| 0.100 | 1.513 | 1.519 | 1.662 | 1.656 | 1.757 | 1.732 | 1.822 | 1.781 | 1.869 | 1.815 | 1.904 | 1.840 | 1.985 | 1.896 | 2.041 | 1.944 | 2.050 | 1.974 |
| 0.200 | 1.342 | 1.343 | 1.425 | 1.419 | 1.478 | 1.458 | 1.512 | 1.483 | 1.537 | 1.499 | 1.554 | 1.511 | 1.592 | 1.536 | 1.611 | 1.558 | 1.610 | 1.571 |
| 0.300 | 1.267 | 1.265 | 1.323 | 1.317 | 1.358 | 1.341 | 1.381 | 1.356 | 1.396 | 1.366 | 1.407 | 1.373 | 1.426 | 1.388 | 1.432 | 1.401 | 1.430 | 1.408 |
| 0.400 | 1.223 | 1.219 | 1.264 | 1.257 | 1.290 | 1.274 | 1.306 | 1.284 | 1.316 | 1.291 | 1.323 | 1.296 | 1.333 | 1.306 | 1.331 | 1.314 | 1.330 | 1.319 |
| 0.500 | 1.193 | 1.188 | 1.226 | 1.217 | 1.245 | 1.230 | 1.257 | 1.237 | 1.265 | 1.242 | 1.269 | 1.245 | 1.275 | 1.252 | 1.267 | 1.258 | 1.265 | 1.261 |
| 1.000 | | 1.112 | | 1.123 | | 1.128 | | 1.131 | | 1.132 | | 1.133 | | 1.136 | | 1.137 | 1.139 | 1.138 |
| 2.000 | | 1.063 | | 1.067 | | 1.068 | | 1.069 | | 1.070 | | 1.070 | | 1.071 | | 1.071 | | 1.071 |
| 10.000 | | 1.014 | | 1.014 | | 1.015 | | 1.015 | | 1.015 | | 1.015 | | 1.015 | | 1.015 | | 1.015 |
| 20.000 | | 1.007 | | 1.007 | | 1.007 | | 1.007 | | 1.007 | | 1.007 | | 1.007 | | 1.007 | | 1.007 |

$\sigma_n = 32M/(\pi d^3)$; $M$ = magnitude of external bending moment; $d$ = diameter of minimum section

**Table 4** $K_t$ of 60° V-shaped circumferential notched round bar under torsion

| | $t/d = 0.01$ | | $t/d = 0.02$ | | $t/d = 0.03$ | | $t/d = 0.04$ | | $t/d = 0.05$ | | $t/d = 0.06$ | | $t/d = 0.10$ | | $t/d = 0.20$ | | $t/d = 0.50$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho/d$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ |
| 0.001 | 4.131 | 3.929 | 5.225 | 4.899 | 5.952 | 5.512 | 6.489 | 5.952 | | 6.287 | | 6.551 | | 7.228 | | 7.937 | | 8.499 |
| 0.002 | 3.193 | 3.072 | 3.963 | 3.743 | 4.470 | 4.168 | 4.841 | 4.472 | | 4.703 | | 4.884 | | 5.346 | | 5.825 | | 6.203 |
| 0.005 | 2.343 | 2.302 | 2.832 | 2.716 | 3.146 | 2.975 | 3.372 | 3.158 | | 3.296 | | 3.404 | | 3.676 | | 3.954 | | 4.168 |
| 0.010 | 1.912 | 1.914 | 2.248 | 2.198 | 2.468 | 2.373 | 2.624 | 2.496 | | 2.587 | | 2.658 | | 2.835 | | 3.012 | | 3.146 |
| 0.020 | 1.623 | 1.639 | 1.837 | 1.831 | 1.984 | 1.946 | 2.092 | 2.026 | 2.175 | 2.085 | 2.240 | 2.130 | 2.404 | 2.241 | 2.546 | 2.349 | 2.604 | 2.429 |
| 0.030 | 1.499 | 1.517 | 1.656 | 1.667 | 1.771 | 1.757 | 1.854 | 1.818 | 1.918 | 1.862 | 1.969 | 1.896 | 2.098 | 1.978 | 2.212 | 2.057 | 2.257 | 2.115 |
| 0.040 | 1.425 | 1.444 | 1.551 | 1.570 | 1.645 | 1.644 | 1.713 | 1.694 | 1.765 | 1.730 | 1.807 | 1.757 | 1.914 | 1.822 | 2.012 | 1.884 | 2.050 | 1.929 |
| 0.050 | 1.375 | 1.394 | 1.484 | 1.503 | 1.563 | 1.567 | 1.620 | 1.609 | 1.664 | 1.639 | 1.700 | 1.662 | 1.790 | 1.716 | 1.875 | 1.767 | 1.909 | 1.804 |
| 0.060 | 1.338 | 1.357 | 1.435 | 1.454 | 1.504 | 1.509 | 1.553 | 1.546 | 1.590 | 1.572 | 1.620 | 1.592 | 1.701 | 1.638 | 1.774 | 1.681 | 1.805 | 1.712 |
| 0.080 | 1.310 | 1.328 | 1.397 | 1.416 | 1.459 | 1.465 | 1.503 | 1.497 | 1.537 | 1.520 | 1.563 | 1.538 | 1.631 | 1.578 | 1.697 | 1.615 | 1.724 | 1.641 |
| 0.100 | 1.252 | 1.269 | 1.320 | 1.337 | 1.368 | 1.374 | 1.401 | 1.398 | 1.427 | 1.415 | 1.447 | 1.428 | 1.498 | 1.456 | 1.544 | 1.482 | 1.558 | 1.500 |
| 0.200 | 1.165 | 1.180 | 1.204 | 1.219 | 1.230 | 1.239 | 1.248 | 1.251 | 1.262 | 1.260 | 1.272 | 1.266 | 1.299 | 1.279 | 1.324 | 1.291 | 1.332 | 1.298 |
| 0.300 | 1.127 | 1.140 | 1.154 | 1.167 | 1.171 | 1.180 | 1.182 | 1.188 | 1.191 | 1.193 | 1.197 | 1.197 | 1.213 | 1.205 | 1.228 | 1.211 | 1.237 | 1.216 |
| 0.400 | 1.104 | 1.116 | 1.126 | 1.136 | 1.138 | 1.146 | 1.146 | 1.151 | 1.151 | 1.155 | 1.156 | 1.157 | 1.165 | 1.163 | 1.174 | 1.167 | 1.182 | 1.170 |
| 0.500 | 1.089 | 1.100 | 1.108 | 1.116 | 1.118 | 1.123 | 1.124 | 1.127 | 1.128 | 1.129 | 1.131 | 1.131 | 1.137 | 1.135 | 1.140 | 1.138 | 1.146 | 1.140 |
| 1.000 | | 1.061 | | 1.067 | | 1.070 | | 1.071 | | 1.072 | | 1.073 | | 1.074 | | 1.075 | 1.078 | 1.076 |
| 2.000 | | 1.035 | | 1.037 | | 1.038 | | 1.038 | | 1.039 | | 1.039 | | 1.039 | | 1.039 | | 1.040 |
| 10.000 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 |
| 20.000 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 |

$\tau_n = 16T/(\pi d^3)$; $T$ = magnitude of external torsional moment; $d$ = diameter of minimum section

**Table 5** $K_t$ of 60° V-shaped double notched flat bar under tension

| | $t/d$ = 0.01 | | $t/d$ = 0.02 | | $t/d$ = 0.03 | | $t/d$ = 0.04 | | $t/d$ = 0.05 | | $t/d$ = 0.06 | | $t/d$ = 0.10 | | $t/d$ = 0.20 | | $t/d$ = 0.50 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho/d$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ |
| 0.001 | 7.847 | 7.791 | 10.559 | 10.426 | 12.588 | 12.273 | 14.168 | 13.701 | 15.483 | 14.861 | 16.606 | 15.831 | 19.922 | 18.565 | 24.538 | 21.968 | 31.150 | 25.241 |
| 0.002 | 5.760 | 5.727 | 7.692 | 7.581 | 9.091 | 8.881 | 10.202 | 9.887 | 11.127 | 10.702 | 11.917 | 11.383 | 14.248 | 13.299 | 17.491 | 15.669 | 22.137 | 17.929 |
| 0.005 | 3.924 | 3.911 | 5.127 | 5.069 | 5.999 | 5.882 | 6.693 | 6.509 | 7.271 | 7.017 | 7.764 | 7.441 | 9.219 | 8.627 | 11.240 | 10.079 | 14.142 | 11.440 |
| 0.010 | 3.004 | 3.000 | 3.847 | 3.814 | 4.453 | 4.379 | 4.935 | 4.815 | 5.337 | 5.167 | 5.680 | 5.460 | 6.691 | 6.276 | 8.094 | 7.262 | 10.115 | 8.170 |
| 0.020 | 2.375 | 2.377 | 2.944 | 2.929 | 3.364 | 3.324 | 3.692 | 3.624 | 3.961 | 3.865 | 4.190 | 4.065 | 4.851 | 4.616 | 5.688 | 5.271 | 6.402 | 5.859 |
| 0.030 | 2.102 | 2.105 | 2.557 | 2.548 | 2.883 | 2.856 | 3.148 | 3.099 | 3.363 | 3.291 | 3.546 | 3.449 | 4.073 | 3.883 | 4.736 | 4.390 | 5.291 | 4.837 |
| 0.040 | 1.942 | 1.946 | 2.328 | 2.321 | 2.606 | 2.584 | 2.824 | 2.784 | 3.009 | 2.949 | 3.165 | 3.083 | 3.612 | 3.446 | 4.170 | 3.866 | 4.630 | 4.229 |
| 0.050 | 1.835 | 1.840 | 2.174 | 2.168 | 2.419 | 2.399 | 2.612 | 2.575 | 2.770 | 2.715 | 2.908 | 2.833 | 3.300 | 3.149 | 3.784 | 3.509 | 4.179 | 3.816 |
| 0.060 | 1.757 | 1.762 | 2.061 | 2.056 | 2.282 | 2.263 | 2.455 | 2.421 | 2.598 | 2.546 | 2.719 | 2.648 | 3.072 | 2.930 | 3.501 | 3.246 | 3.848 | 3.512 |
| 0.080 | 1.696 | 1.701 | 1.974 | 1.970 | 2.176 | 2.158 | 2.334 | 2.301 | 2.465 | 2.415 | 2.574 | 2.507 | 2.894 | 2.760 | 3.282 | 3.042 | 3.590 | 3.276 |
| 0.100 | 1.575 | 1.580 | 1.800 | 1.796 | 1.962 | 1.945 | 2.089 | 2.058 | 2.194 | 2.147 | 2.282 | 2.220 | 2.531 | 2.413 | 2.836 | 2.628 | 3.066 | 2.800 |
| 0.200 | 1.397 | 1.400 | 1.543 | 1.539 | 1.648 | 1.631 | 1.728 | 1.699 | 1.793 | 1.752 | 1.848 | 1.794 | 1.999 | 1.903 | 2.173 | 2.017 | 2.284 | 2.104 |
| 0.300 | 1.319 | 1.321 | 1.431 | 1.427 | 1.511 | 1.494 | 1.572 | 1.543 | 1.620 | 1.580 | 1.660 | 1.609 | 1.768 | 1.682 | 1.888 | 1.757 | 1.948 | 1.811 |
| 0.400 | 1.273 | 1.273 | 1.365 | 1.359 | 1.431 | 1.413 | 1.480 | 1.450 | 1.518 | 1.478 | 1.550 | 1.500 | 1.633 | 1.554 | 1.722 | 1.607 | 1.754 | 1.645 |
| 0.500 | 1.242 | 1.241 | 1.321 | 1.314 | 1.376 | 1.358 | 1.418 | 1.388 | 1.450 | 1.410 | 1.475 | 1.427 | 1.543 | 1.469 | 1.611 | 1.509 | 1.625 | 1.537 |
| 1.000 | | 1.160 | | 1.200 | | 1.222 | | 1.236 | | 1.246 | | 1.253 | | 1.270 | | 1.285 | 1.326 | 1.295 |
| 2.000 | | 1.102 | | 1.121 | | 1.130 | | 1.136 | | 1.140 | | 1.142 | | 1.148 | | 1.153 | | 1.156 |
| 10.000 | | 1.029 | | 1.031 | | 1.031 | | 1.032 | | 1.032 | | 1.032 | | 1.032 | | 1.033 | | 1.033 |
| 20.000 | | 1.015 | | 1.016 | | 1.016 | | 1.016 | | 1.016 | | 1.016 | | 1.016 | | 1.017 | | 1.017 |

$\sigma_n = P/dh$; $P$ = magnitude of external load; $d$ = width of minimum section; $h$ = plate thickness

**Table 6** $K_t$ of 60° V-shaped double notched flat bar under in-plane bending

| | $t/d$ = 0.01 | | $t/d$ = 0.02 | | $t/d$ = 0.03 | | $t/d$ = 0.04 | | $t/d$ = 0.05 | | $t/d$ = 0.06 | | $t/d$ = 0.10 | | $t/d$ = 0.20 | | $t/d$ = 0.50 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho/d$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ |
| 0.001 | 7.600 | 7.532 | 9.992 | 9.767 | 11.592 | 11.193 | 12.722 | 12.212 | 13.676 | 12.986 | | 13.597 | | 15.147 | | 16.752 | | 18.007 |
| 0.002 | 5.578 | 5.543 | 7.252 | 7.110 | 8.371 | 8.107 | 9.197 | 8.818 | 9.828 | 9.356 | | 9.780 | | 10.849 | | 11.947 | | 12.796 |
| 0.005 | 3.800 | 3.793 | 4.834 | 4.763 | 5.525 | 5.378 | 6.034 | 5.814 | 6.422 | 6.142 | | 6.399 | | 7.042 | | 7.691 | | 8.183 |
| 0.010 | 2.909 | 2.915 | 3.626 | 3.590 | 4.101 | 4.011 | 4.449 | 4.307 | 4.714 | 4.529 | | 4.701 | | 5.129 | | 5.553 | | 5.869 |
| 0.020 | 2.301 | 2.313 | 2.776 | 2.764 | 3.097 | 3.051 | 3.328 | 3.249 | 3.505 | 3.395 | 3.646 | 3.508 | 4.009 | 3.784 | 4.366 | 4.052 | 4.547 | 4.246 |
| 0.030 | 2.036 | 2.051 | 2.410 | 2.407 | 2.657 | 2.627 | 2.840 | 2.783 | 2.978 | 2.896 | 3.087 | 2.983 | 3.365 | 3.193 | 3.638 | 3.393 | 3.773 | 3.536 |
| 0.040 | 1.881 | 1.897 | 2.194 | 2.195 | 2.401 | 2.379 | 2.550 | 2.505 | 2.666 | 2.600 | 2.757 | 2.672 | 2.985 | 2.843 | 3.206 | 3.003 | 3.316 | 3.116 |
| 0.050 | 1.777 | 1.793 | 2.049 | 2.052 | 2.229 | 2.211 | 2.358 | 2.319 | 2.455 | 2.398 | 2.534 | 2.460 | 2.729 | 2.605 | 2.914 | 2.740 | 3.005 | 2.833 |
| 0.060 | 1.701 | 1.717 | 1.942 | 1.947 | 2.102 | 2.087 | 2.216 | 2.183 | 2.302 | 2.252 | 2.369 | 2.304 | 2.542 | 2.431 | 2.701 | 2.547 | 2.777 | 2.626 |
| 0.800 | 1.596 | 1.612 | 1.796 | 1.800 | 1.927 | 1.914 | 2.021 | 1.992 | 2.091 | 2.047 | 2.146 | 2.089 | 2.282 | 2.189 | 2.405 | 2.279 | 2.460 | 2.339 |
| 0.100 | 1.526 | 1.540 | 1.697 | 1.702 | 1.809 | 1.798 | 1.888 | 1.862 | 1.948 | 1.909 | 1.994 | 1.944 | 2.107 | 2.025 | 2.208 | 2.099 | 2.247 | 2.147 |
| 0.200 | 1.353 | 1.364 | 1.456 | 1.460 | 1.521 | 1.514 | 1.565 | 1.548 | 1.598 | 1.573 | 1.624 | 1.591 | 1.684 | 1.632 | 1.732 | 1.667 | 1.745 | 1.690 |
| 0.300 | 1.278 | 1.287 | 1.351 | 1.355 | 1.396 | 1.391 | 1.427 | 1.414 | 1.449 | 1.429 | 1.466 | 1.441 | 1.503 | 1.467 | 1.530 | 1.488 | 1.535 | 1.502 |
| 0.400 | 1.233 | 1.240 | 1.290 | 1.293 | 1.324 | 1.319 | 1.347 | 1.336 | 1.363 | 1.347 | 1.375 | 1.355 | 1.401 | 1.373 | 1.415 | 1.388 | 1.416 | 1.397 |
| 0.500 | 1.203 | 1.209 | 1.250 | 1.251 | 1.277 | 1.271 | 1.295 | 1.284 | 1.307 | 1.292 | 1.316 | 1.298 | 1.334 | 1.311 | 1.341 | 1.322 | 1.338 | 1.329 |
| 1.000 | | 1.130 | | 1.150 | | 1.158 | | 1.163 | | 1.166 | | 1.168 | | 1.173 | | 1.177 | 1.178 | 1.179 |
| 2.000 | | 1.077 | | 1.085 | | 1.088 | | 1.089 | | 1.090 | | 1.091 | | 1.092 | | 1.093 | | 1.094 |
| 10.000 | | 1.019 | | 1.019 | | 1.019 | | 1.019 | | 1.020 | | 1.020 | | 1.020 | | 1.020 | | 1.020 |
| 20.000 | | 1.010 | | 1.010 | | 1.010 | | 1.010 | | 1.010 | | 1.010 | | 1.010 | | 1.010 | | 1.010 |

$\sigma_n = 6M/dh^2$; $M$ = magnitude of external bending moment; $d$ = width of minimum section; $h$ = plate thickness

**Table 7** $K_t$ of 60° V-shaped double notched flat bar under transverse bending

| | $t/d$ = 0.01 | | $t/d$ = 0.02 | | $t/d$ = 0.03 | | $t/d$ = 0.04 | | $t/d$ = 0.05 | | $t/d$ = 0.06 | | $t/d$ = 0.10 | | $t/d$ = 0.20 | | $t/d$ = 0.50 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho/d$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ |
| 0.001 | 3.563 | 3.426 | 4.469 | 4.344 | 5.068 | 4.998 | 5.523 | 5.511 | 5.889 | 5.933 | 6.195 | 6.291 | 7.066 | 7.327 | 8.191 | 8.686 | | 10.093 |
| 0.002 | 2.791 | 2.713 | 3.498 | 3.359 | 3.966 | 3.817 | 4.315 | 4.176 | 4.592 | 4.471 | 4.820 | 4.720 | 5.455 | 5.437 | 6.240 | 6.369 | | 7.322 |
| 0.005 | 2.111 | 2.081 | 2.540 | 2.485 | 2.868 | 2.769 | 3.121 | 2.991 | 3.321 | 3.172 | 3.486 | 3.324 | 3.931 | 3.758 | 4.443 | 4.312 | | 4.864 |
| 0.010 | 1.754 | 1.762 | 2.072 | 2.044 | 2.276 | 2.241 | 2.453 | 2.393 | 2.602 | 2.516 | 2.728 | 2.619 | 3.077 | 2.911 | 3.471 | 3.274 | | 3.626 |
| 0.020 | 1.512 | 1.536 | 1.723 | 1.731 | 1.876 | 1.865 | 1.996 | 1.968 | 2.095 | 2.051 | 2.179 | 2.119 | 2.417 | 2.309 | 2.702 | 2.539 | 2.947 | 2.752 |
| 0.030 | 1.409 | 1.436 | 1.570 | 1.592 | 1.690 | 1.698 | 1.785 | 1.779 | 1.864 | 1.844 | 1.931 | 1.897 | 2.124 | 2.041 | 2.359 | 2.212 | 2.545 | 2.367 |
| 0.040 | 1.348 | 1.376 | 1.482 | 1.509 | 1.581 | 1.599 | 1.660 | 1.666 | 1.726 | 1.720 | 1.782 | 1.763 | 1.947 | 1.881 | 2.152 | 2.018 | 2.306 | 2.138 |
| 0.050 | 1.308 | 1.335 | 1.426 | 1.452 | 1.511 | 1.530 | 1.579 | 1.589 | 1.635 | 1.634 | 1.683 | 1.672 | 1.826 | 1.772 | 2.007 | 1.885 | 2.143 | 1.983 |
| 0.060 | 1.278 | 1.304 | 1.384 | 1.410 | 1.460 | 1.479 | 1.519 | 1.531 | 1.567 | 1.571 | 1.609 | 1.604 | 1.735 | 1.691 | 1.897 | 1.788 | 2.022 | 1.870 |
| 0.080 | 1.255 | 1.281 | 1.352 | 1.377 | 1.421 | 1.440 | 1.474 | 1.486 | 1.518 | 1.522 | 1.555 | 1.551 | 1.665 | 1.628 | 1.808 | 1.712 | 1.926 | 1.782 |
| 0.100 | 1.209 | 1.232 | 1.287 | 1.309 | 1.341 | 1.359 | 1.383 | 1.394 | 1.417 | 1.422 | 1.446 | 1.443 | 1.530 | 1.499 | 1.637 | 1.558 | 1.720 | 1.605 |
| 0.200 | 1.140 | 1.159 | 1.189 | 1.206 | 1.222 | 1.235 | 1.246 | 1.254 | 1.265 | 1.269 | 1.281 | 1.280 | 1.326 | 1.307 | 1.385 | 1.333 | 1.427 | 1.352 |
| 0.300 | 1.111 | 1.126 | 1.147 | 1.160 | 1.169 | 1.179 | 1.186 | 1.192 | 1.198 | 1.201 | 1.209 | 1.208 | 1.238 | 1.224 | 1.275 | 1.239 | 1.302 | 1.249 |
| 0.400 | 1.093 | 1.105 | 1.122 | 1.131 | 1.140 | 1.146 | 1.152 | 1.155 | 1.161 | 1.161 | 1.168 | 1.166 | 1.188 | 1.176 | 1.213 | 1.186 | 1.230 | 1.193 |
| 0.500 | 1.0081 | 1.091 | 1.107 | 1.112 | 1.121 | 1.123 | 1.131 | 1.130 | 1.138 | 1.134 | 1.143 | 1.138 | 1.158 | 1.146 | 1.173 | 1.152 | 1.182 | 1.157 |
| 1.000 | | 1.056 | | 1.065 | | 1.069 | | 1.072 | | 1.073 | | 1.074 | | 1.077 | | 1.079 | 1.089 | 1.080 |
| 2.000 | | 1.032 | | 1.036 | | 1.037 | | 1.038 | | 1.038 | | 1.038 | | 1.039 | | 1.039 | | 1.040 |
| 10.000 | | 1.007 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 |
| 20.000 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 |

$\sigma_n = 6M/dh^2$; $M$ = magnitude of external bending moment; $d$ = width of minimum section; $h$ = plate thickness

**Table 8** $K_t$ of U-shaped double notched flat bar under transverse bending

| | $t/d$ = 0.01 | | $t/d$ = 0.02 | | $t/d$ = 0.03 | | $t/d$ = 0.04 | | $t/d$ = 0.05 | | $t/d$ = 0.06 | | $t/d$ = 0.10 | | $t/d$ = 0.20 | | $t/d$ = 0.50 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho/d$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ |
| 0.001 | 3.849 | 3.426 | 5.118 | 4.344 | 6.047 | 4.998 | 6.791 | 5.511 | 7.413 | 5.933 | 7.947 | 6.291 | 9.531 | 7.327 | 11.694 | 8.686 | | 10.093 |
| 0.002 | 2.910 | 2.713 | 3.778 | 3.359 | 4.421 | 3.817 | 4.938 | 4.176 | 5.372 | 4.471 | 5.745 | 4.720 | 6.851 | 5.437 | 8.363 | 6.369 | | 7.322 |
| 0.005 | 2.125 | 2.081 | 2.629 | 2.485 | 3.011 | 2.769 | 3.321 | 2.991 | 3.584 | 3.172 | 3.810 | 3.324 | 4.486 | 3.758 | 5.413 | 4.312 | | 4.864 |
| 0.010 | 1.756 | 1.762 | 2.086 | 2.044 | 2.333 | 2.241 | 2.537 | 2.393 | 2.710 | 2.516 | 2.860 | 2.619 | 3.312 | 2.911 | 3.936 | 3.274 | | 3.626 |
| 0.020 | 1.512 | 1.536 | 1.728 | 1.731 | 1.888 | 1.865 | 2.018 | 1.968 | 2.129 | 2.051 | 2.225 | 2.119 | 2.519 | 2.309 | 2.932 | 2.539 | 3.357 | 2.752 |
| 0.030 | 1.408 | 1.436 | 1.569 | 1.592 | 1.689 | 1.698 | 1.787 | 1.779 | 1.870 | 1.844 | 1.942 | 1.897 | 2.166 | 2.041 | 2.486 | 2.212 | 2.823 | 2.367 |
| 0.040 | 1.348 | 1.376 | 1.478 | 1.509 | 1.576 | 1.599 | 1.654 | 1.666 | 1.721 | 1.720 | 1.779 | 1.763 | 1.959 | 1.881 | 2.220 | 2.018 | 2.499 | 2.138 |
| 0.050 | 1.307 | 1.335 | 1.425 | 1.452 | 1.509 | 1.530 | 1.576 | 1.589 | 1.631 | 1.634 | 1.680 | 1.672 | 1.828 | 1.772 | 2.043 | 1.885 | 2.278 | 1.983 |
| 0.060 | 1.277 | 1.304 | 1.384 | 1.410 | 1.460 | 1.479 | 1.519 | 1.531 | 1.569 | 1.571 | 1.611 | 1.604 | 1.745 | 1.691 | 1.921 | 1.788 | 2.116 | 1.870 |
| 0.080 | 1.255 | 1.281 | 1.352 | 1.377 | 1.421 | 1.440 | 1.475 | 1.486 | 1.519 | 1.522 | 1.557 | 1.551 | 1.672 | 1.628 | 1.836 | 1.712 | 1.994 | 1.782 |
| 0.100 | 1.208 | 1.232 | 1.287 | 1.309 | 1.342 | 1.359 | 1.384 | 1.394 | 1.418 | 1.422 | 1.448 | 1.443 | 1.535 | 1.499 | 1.653 | 1.558 | 1.767 | 1.605 |
| 0.200 | 1.140 | 1.159 | 1.189 | 1.206 | 1.222 | 1.235 | 1.246 | 1.254 | 1.265 | 1.269 | 1.281 | 1.280 | 1.327 | 1.307 | 1.390 | 1.333 | 1.447 | 1.352 |
| 0.300 | 1.111 | 1.126 | 1.147 | 1.160 | 1.169 | 1.179 | 1.186 | 1.192 | 1.198 | 1.201 | 1.208 | 1.208 | 1.237 | 1.224 | 1.275 | 1.239 | 1.311 | 1.249 |
| 0.400 | 1.093 | 1.105 | 1.122 | 1.131 | 1.140 | 1.146 | 1.152 | 1.155 | 1.161 | 1.161 | 1.168 | 1.166 | 1.187 | 1.176 | 1.209 | 1.186 | 1.231 | 1.193 |
| 0.500 | 1.081 | 1.091 | 1.107 | 1.112 | 1.121 | 1.123 | 1.131 | 1.130 | 1.138 | 1.134 | 1.143 | 1.138 | 1.156 | 1.146 | 1.168 | 1.152 | 1.179 | 1.157 |
| 1.000 | | 1.056 | | 1.065 | | 1.069 | | 1.072 | | 1.073 | | 1.074 | | 1.077 | | 1.079 | 1.089 | 1.080 |
| 2.000 | | 1.032 | | 1.036 | | 1.037 | | 1.038 | | 1.038 | | 1.038 | | 1.039 | | 1.039 | | 1.040 |
| 10.000 | | 1.007 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 |
| 20.000 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 |

$\sigma_n = 6M/dh^2$; $M$ = magnitude of external bending moment; $d$ = width of minimum section; $h$ = plate thickness

**Table 9** $K_t$ of 60° V-shaped single notched flat bar under pure tension (no bending moment at the minimum section)

| | $t/d$ = 0.01 | | $t/d$ = 0.02 | | $t/d$ = 0.03 | | $t/d$ = 0.04 | | $t/d$ = 0.05 | | $t/d$ = 0.06 | | $t/d$ = 0.10 | | $t/d$ = 0.20 | | $t/d$ = 0.50 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho/d$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ |
| 0.001 | 7.433 | 7.121 | 9.563 | 8.841 | 10.875 | 9.812 | 10.445 | 10.445 | 12.432 | 10.894 | 12.927 | 11.229 | 14.031 | 12.009 | | 12.717 | | 13.206 |
| 0.002 | 5.456 | 5.244 | 6.941 | 6.434 | 7.854 | 7.100 | 7.532 | 7.532 | 8.934 | 7.836 | 9.277 | 8.062 | 10.035 | 8.584 | | 9.052 | | 9.373 |
| 0.005 | 3.717 | 3.589 | 4.626 | 4.305 | 5.183 | 4.698 | 5.563 | 4.950 | 5.838 | 5.125 | 6.044 | 5.254 | 6.493 | 5.548 | | 5.806 | | 5.979 |
| 0.010 | 2.845 | 2.759 | 3.471 | 3.237 | 3.847 | 3.492 | 4.102 | 3.652 | 4.285 | 3.762 | 4.421 | 3.843 | 4.712 | 4.024 | | 4.179 | | 4.281 |
| 0.020 | 2.250 | 2.185 | 2.661 | 2.484 | 2.911 | 2.644 | 3.078 | 2.740 | 3.196 | 2.805 | 3.283 | 2.852 | 3.460 | 2.956 | 3.537 | 3.042 | 3.532 | 3.097 |
| 0.030 | 1.991 | 1.933 | 2.312 | 2.157 | 2.503 | 2.270 | 2.633 | 2.340 | 2.725 | 2.386 | 2.791 | 2.419 | 2.926 | 2.490 | 2.982 | 2.548 | 2.976 | 2.585 |
| 0.040 | 1.840 | 1.785 | 2.106 | 1.962 | 2.265 | 2.051 | 2.371 | 2.103 | 2.447 | 2.139 | 2.502 | 2.164 | 2.611 | 2.218 | 2.656 | 2.261 | 2.653 | 2.287 |
| 0.050 | 1.738 | 1.684 | 1.969 | 1.831 | 2.105 | 1.903 | 2.196 | 1.945 | 2.259 | 1.973 | 2.306 | 1.993 | 2.398 | 2.035 | 2.435 | 2.069 | 2.437 | 2.089 |
| 0.060 | 1.664 | 1.611 | 1.869 | 1.734 | 1.989 | 1.794 | 2.069 | 1.830 | 2.125 | 1.852 | 2.164 | 1.868 | 2.240 | 1.903 | 2.272 | 1.930 | 2.280 | 1.946 |
| 0.080 | 1.607 | 1.554 | 1.789 | 1.660 | 1.896 | 1.711 | 1.967 | 1.741 | 2.017 | 1.760 | 2.052 | 1.774 | 2.119 | 1.802 | 2.144 | 1.825 | 2.158 | 1.838 |
| 0.100 | 1.492 | 1.438 | 1.630 | 1.512 | 1.711 | 1.546 | 1.765 | 1.565 | 1.801 | 1.578 | 1.827 | 1.586 | 1.874 | 1.604 | 1.890 | 1.618 | 1.902 | 1.627 |
| 0.200 | 1.323 | 1.269 | 1.404 | 1.301 | 1.450 | 1.315 | 1.478 | 1.322 | 1.497 | 1.327 | 1.509 | 1.330 | 1.528 | 1.337 | 1.531 | 1.342 | 1.539 | 1.345 |
| 0.300 | 1.250 | 1.197 | 1.310 | 1.216 | 1.341 | 1.223 | 1.360 | 1.227 | 1.372 | 1.230 | 1.379 | 1.232 | 1.385 | 1.235 | 1.384 | 1.238 | 1.391 | 1.239 |
| 0.400 | 1.206 | 1.157 | 1.254 | 1.169 | 1.278 | 1.174 | 1.291 | 1.176 | 1.299 | 1.178 | 1.303 | 1.179 | 1.303 | 1.181 | 1.301 | 1.183 | 1.306 | 1.184 |
| 0.500 | 1.176 | 1.131 | 1.214 | 1.139 | 1.233 | 1.143 | 1.243 | 1.144 | 1.248 | 1.145 | 1.251 | 1.146 | 1.249 | 1.148 | 1.246 | 1.149 | 1.249 | 1.149 |
| 1.000 | | 1.072 | | 1.075 | | 1.076 | | 1.076 | | 1.077 | | 1.077 | | 1.077 | | 1.078 | 1.129 | 1.078 |
| 2.000 | | 1.038 | | 1.039 | | 1.040 | | 1.040 | | 1.040 | | 1.040 | | 1.040 | | 1.040 | | 1.040 |
| 10.000 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 |
| 20.000 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 |

$\sigma_n = P/dh$; $P$ = magnitude of external load; $d$ = width of minimum section; $h$ = plate thickness

**Table 10** $K_t$ of 60° V-shaped single notched flat bar under in-plane bending

| | $t/d$ = 0.01 | | $t/d$ = 0.02 | | $t/d$ = 0.03 | | $t/d$ = 0.04 | | $t/d$ = 0.05 | | $t/d$ = 0.06 | | $t/d$ = 0.10 | | $t/d$ = 0.20 | | $t/d$ = 0.50 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho/d$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ |
| 0.001 | 7.528 | 7.494 | 9.813 | 9.676 | 11.300 | 11.051 | 12.374 | 12.025 | 13.187 | 12.758 | | 13.332 | | 14.775 | | 16.244 | | 17.373 |
| 0.002 | 5.526 | 5.517 | 7.122 | 7.047 | 8.161 | 8.008 | 8.911 | 8.687 | 9.477 | 9.197 | | 9.595 | | 10.591 | | 11.595 | | 12.359 |
| 0.005 | 3.765 | 3.777 | 4.747 | 4.725 | 5.386 | 5.318 | 5.846 | 5.735 | 6.193 | 6.046 | | 6.288 | | 6.887 | | 7.483 | | 7.928 |
| 0.010 | 2.882 | 2.905 | 3.561 | 3.565 | 3.997 | 3.972 | 4.311 | 4.256 | 4.545 | 4.467 | | 4.630 | | 5.031 | | 5.423 | | 5.711 |
| 0.020 | 2.279 | 2.307 | 2.705 | 2.750 | 2.988 | 3.029 | 3.187 | 3.219 | 3.336 | 3.360 | 3.453 | 3.467 | 3.733 | 3.729 | 3.949 | 3.980 | 3.998 | 4.160 |
| 0.030 | 2.017 | 2.047 | 2.347 | 2.398 | 2.561 | 2.612 | 2.715 | 2.764 | 2.829 | 2.873 | 2.918 | 2.957 | 3.128 | 3.158 | 3.286 | 3.348 | 3.315 | 3.482 |
| 0.040 | 1.863 | 1.894 | 2.136 | 2.189 | 2.313 | 2.370 | 2.436 | 2.492 | 2.530 | 2.585 | 2.602 | 2.655 | 2.771 | 2.820 | 2.893 | 2.974 | 2.910 | 3.082 |
| 0.050 | 1.760 | 1.791 | 1.995 | 2.048 | 2.148 | 2.204 | 2.253 | 2.311 | 2.330 | 2.388 | 2.391 | 2.449 | 2.531 | 2.591 | 2.628 | 2.721 | 2.636 | 2.811 |
| 0.060 | 1.685 | 1.716 | 1.892 | 1.944 | 2.025 | 2.083 | 2.117 | 2.178 | 2.184 | 2.246 | 2.235 | 2.298 | 2.359 | 2.423 | 2.435 | 2.536 | 2.436 | 2.613 |
| 0.080 | 1.628 | 1.658 | 1.813 | 1.864 | 1.933 | 1.989 | 2.015 | 2.074 | 2.075 | 2.136 | 2.120 | 2.182 | 2.223 | 2.292 | 2.289 | 2.393 | 2.282 | 2.461 |
| 0.100 | 1.511 | 1.541 | 1.654 | 1.703 | 1.746 | 1.800 | 1.807 | 1.865 | 1.852 | 1.912 | 1.886 | 1.947 | 1.959 | 2.029 | 1.998 | 2.104 | 1.981 | 2.153 |
| 0.200 | 1.340 | 1.367 | 1.419 | 1.465 | 1.469 | 1.521 | 1.500 | 1.557 | 1.522 | 1.583 | 1.538 | 1.602 | 1.570 | 1.645 | 1.581 | 1.683 | 1.565 | 1.708 |
| 0.300 | 1.266 | 1.290 | 1.316 | 1.361 | 1.349 | 1.400 | 1.368 | 1.424 | 1.380 | 1.441 | 1.389 | 1.454 | 1.403 | 1.482 | 1.404 | 1.506 | 1.396 | 1.521 |
| 0.400 | 1.221 | 1.244 | 1.258 | 1.300 | 1.281 | 1.329 | 1.293 | 1.346 | 1.301 | 1.359 | 1.305 | 1.368 | 1.310 | 1.388 | 1.306 | 1.404 | 1.301 | 1.415 |
| 0.500 | 1.191 | 1.213 | 1.220 | 1.258 | 1.238 | 1.281 | 1.247 | 1.294 | 1.251 | 1.304 | 1.254 | 1.311 | 1.254 | 1.326 | 1.245 | 1.338 | 1.241 | 1.346 |
| 1.000 | | 1.135 | | 1.156 | | 1.166 | | 1.172 | | 1.175 | | 1.178 | | 1.183 | | 1.188 | 1.127 | 1.191 |
| 2.000 | | 1.081 | | 1.089 | | 1.093 | | 1.095 | | 1.096 | | 1.097 | | 1.099 | | 1.100 | | 1.101 |
| 10.000 | | 1.020 | | 1.021 | | 1.021 | | 1.021 | | 1.021 | | 1.021 | | 1.021 | | 1.021 | | 1.021 |
| 20.000 | | 1.010 | | 1.011 | | 1.011 | | 1.011 | | 1.011 | | 1.011 | | 1.011 | | 1.011 | | 1.011 |

$\sigma_n = 6M/dh^2$; $M$ = magnitude of external bending moment; $d$ = width of minimum section; $h$ = plate thickness

**Table 11** $K_t$ of 60° V-shaped single notched flat bar under transverse bending

| | $t/d = 0.01$ | | $t/d = 0.02$ | | $t/d = 0.03$ | | $t/d = 0.04$ | | $t/d = 0.05$ | | $t/d = 0.06$ | | $t/d = 0.10$ | | $t/d = 0.20$ | | $t/d = 0.50$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho/d$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ |
| 0.001 | 3.562 | 3.426 | 4.467 | 4.344 | 5.066 | 4.998 | 5.520 | 5.511 | 5.886 | 5.933 | 6.193 | 6.291 | 7.069 | 7.327 | 8.214 | 8.686 | 9.460 | 10.093 |
| 0.002 | 2.790 | 2.713 | 3.497 | 3.359 | 3.964 | 3.817 | 4.313 | 4.176 | 4.590 | 4.471 | 4.819 | 4.720 | 5.458 | 5.437 | 6.257 | 6.369 | 7.073 | 7.322 |
| 0.005 | 2.110 | 2.081 | 2.539 | 2.485 | 2.867 | 2.769 | 3.119 | 2.991 | 3.320 | 3.172 | 3.485 | 3.324 | 3.933 | 3.758 | 4.455 | 4.312 | 4.921 | 4.864 |
| 0.010 | 1.753 | 1.762 | 2.071 | 2.044 | 2.275 | 2.241 | 2.452 | 2.393 | 2.601 | 2.516 | 2.727 | 2.619 | 3.078 | 2.911 | 3.481 | 3.274 | 3.799 | 3.626 |
| 0.020 | 1.512 | 1.536 | 1.724 | 1.731 | 1.877 | 1.865 | 1.998 | 1.968 | 2.097 | 2.051 | 2.181 | 2.119 | 2.418 | 2.309 | 2.707 | 2.539 | 2.963 | 2.752 |
| 0.030 | 1.408 | 1.436 | 1.571 | 1.592 | 1.691 | 1.698 | 1.787 | 1.779 | 1.865 | 1.844 | 1.932 | 1.897 | 2.125 | 2.041 | 2.363 | 2.212 | 2.560 | 2.367 |
| 0.040 | 1.348 | 1.376 | 1.482 | 1.509 | 1.582 | 1.599 | 1.662 | 1.666 | 1.727 | 1.720 | 1.784 | 1.763 | 1.948 | 1.881 | 2.156 | 2.018 | 2.321 | 2.138 |
| 0.050 | 1.307 | 1.335 | 1.426 | 1.452 | 1.512 | 1.530 | 1.580 | 1.589 | 1.636 | 1.634 | 1.684 | 1.672 | 1.827 | 1.772 | 2.011 | 1.885 | 2.158 | 1.983 |
| 0.060 | 1.277 | 1.304 | 1.385 | 1.410 | 1.461 | 1.479 | 1.520 | 1.531 | 1.569 | 1.571 | 1.610 | 1.604 | 1.736 | 1.691 | 1.901 | 1.788 | 2.036 | 1.870 |
| 0.080 | 1.255 | 1.281 | 1.353 | 1.377 | 1.422 | 1.440 | 1.475 | 1.486 | 1.519 | 1.522 | 1.557 | 1.551 | 1.666 | 1.628 | 1.813 | 1.712 | 1.939 | 1.782 |
| 0.100 | 1.208 | 1.232 | 1.288 | 1.309 | 1.342 | 1.359 | 1.384 | 1.394 | 1.418 | 1.422 | 1.447 | 1.443 | 1.532 | 1.499 | 1.641 | 1.558 | 1.732 | 1.605 |
| 0.200 | 1.140 | 1.159 | 1.190 | 1.206 | 1.222 | 1.235 | 1.246 | 1.254 | 1.266 | 1.269 | 1.282 | 1.280 | 1.329 | 1.307 | 1.389 | 1.333 | 1.438 | 1.352 |
| 0.300 | 1.110 | 1.126 | 1.147 | 1.160 | 1.170 | 1.179 | 1.186 | 1.192 | 1.199 | 1.201 | 1.210 | 1.208 | 1.241 | 1.224 | 1.281 | 1.239 | 1.313 | 1.249 |
| 0.400 | 1.093 | 1.105 | 1.123 | 1.131 | 1.140 | 1.146 | 1.153 | 1.155 | 1.162 | 1.161 | 1.170 | 1.166 | 1.191 | 1.176 | 1.218 | 1.186 | 1.241 | 1.193 |
| 0.500 | 1.081 | 1.091 | 1.107 | 1.112 | 1.122 | 1.123 | 1.132 | 1.130 | 1.140 | 1.134 | 1.145 | 1.138 | 1.161 | 1.146 | 1.179 | 1.152 | 1.194 | 1.157 |
| 1.000 | | 1.056 | | 1.065 | | 1.069 | | 1.072 | | 1.073 | | 1.074 | | 1.077 | | 1.079 | 1.100 | 1.080 |
| 2.000 | | 1.032 | | 1.036 | | 1.037 | | 1.038 | | 1.038 | | 1.038 | | 1.039 | | 1.039 | | 1.040 |
| 10.000 | | 1.007 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 |
| 20.000 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 |

$\sigma_n = 6M/dh^2$; $M$ = magnitude of external bending moment; $d$ = width of minimum section; $h$ = plate thickness

**Table 12** $K_t$ of U-shaped single-notched flat bar under transverse bending

| | $t/d = 0.01$ | | $t/d = 0.02$ | | $t/d = 0.03$ | | $t/d = 0.04$ | | $t/d = 0.05$ | | $t/d = 0.06$ | | $t/d = 0.10$ | | $t/d = 0.20$ | | $t/d = 0.50$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho/d$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ | $K_t$ | $K_{tN}$ |
| 0.001 | 3.847 | 3.426 | 5.114 | 4.344 | 6.042 | 4.998 | 6.785 | 5.511 | 7.408 | 5.933 | 7.943 | 6.291 | 9.538 | 7.327 | 11.755 | 8.686 | 14.221 | 10.093 |
| 0.002 | 2.909 | 2.713 | 3.775 | 3.359 | 4.417 | 3.817 | 4.934 | 4.176 | 5.368 | 4.471 | 5.741 | 4.720 | 6.856 | 5.437 | 8.406 | 6.369 | 10.119 | 7.322 |
| 0.005 | 2.124 | 2.081 | 2.627 | 2.485 | 3.008 | 2.769 | 3.318 | 2.991 | 3.581 | 3.172 | 3.808 | 3.324 | 4.490 | 3.758 | 5.442 | 4.312 | 6.490 | 4.864 |
| 0.010 | 1.755 | 1.762 | 2.084 | 2.044 | 2.331 | 2.241 | 2.535 | 2.393 | 2.708 | 2.516 | 2.858 | 2.619 | 3.314 | 2.911 | 3.957 | 3.274 | 4.665 | 3.626 |
| 0.020 | 1.512 | 1.536 | 1.728 | 1.731 | 1.888 | 1.865 | 2.019 | 1.968 | 2.130 | 2.051 | 2.226 | 2.119 | 2.521 | 2.309 | 2.936 | 2.539 | 3.367 | 2.752 |
| 0.030 | 1.408 | 1.436 | 1.569 | 1.592 | 1.691 | 1.698 | 1.788 | 1.779 | 1.871 | 1.844 | 1.944 | 1.897 | 2.167 | 2.041 | 2.490 | 2.212 | 2.834 | 2.367 |
| 0.040 | 1.347 | 1.376 | 1.479 | 1.509 | 1.578 | 1.599 | 1.656 | 1.666 | 1.723 | 1.720 | 1.781 | 1.763 | 1.960 | 1.881 | 2.223 | 2.018 | 2.511 | 2.138 |
| 0.050 | 1.307 | 1.335 | 1.425 | 1.452 | 1.510 | 1.530 | 1.577 | 1.589 | 1.633 | 1.634 | 1.681 | 1.672 | 1.829 | 1.772 | 2.046 | 1.885 | 2.290 | 1.983 |
| 0.060 | 1.277 | 1.304 | 1.385 | 1.410 | 1.461 | 1.479 | 1.520 | 1.531 | 1.570 | 1.571 | 1.612 | 1.604 | 1.746 | 1.691 | 1.924 | 1.788 | 2.129 | 1.870 |
| 0.080 | 1.254 | 1.281 | 1.353 | 1.377 | 1.422 | 1.440 | 1.476 | 1.486 | 1.521 | 1.522 | 1.559 | 1.551 | 1.673 | 1.628 | 1.840 | 1.712 | 2.006 | 1.782 |
| 0.100 | 1.208 | 1.232 | 1.288 | 1.309 | 1.342 | 1.359 | 1.385 | 1.394 | 1.419 | 1.422 | 1.449 | 1.443 | 1.537 | 1.499 | 1.657 | 1.558 | 1.777 | 1.605 |
| 0.200 | 1.140 | 1.159 | 1.190 | 1.206 | 1.222 | 1.235 | 1.247 | 1.254 | 1.266 | 1.269 | 1.282 | 1.280 | 1.330 | 1.307 | 1.395 | 1.333 | 1.457 | 1.352 |
| 0.300 | 1.110 | 1.126 | 1.147 | 1.160 | 1.170 | 1.179 | 1.186 | 1.192 | 1.199 | 1.201 | 1.210 | 1.208 | 1.240 | 1.224 | 1.280 | 1.239 | 1.322 | 1.249 |
| 0.400 | 1.092 | 1.105 | 1.123 | 1.131 | 1.140 | 1.146 | 1.153 | 1.155 | 1.162 | 1.161 | 1.169 | 1.166 | 1.189 | 1.176 | 1.215 | 1.186 | 1.242 | 1.193 |
| 0.500 | 1.080 | 1.091 | 1.107 | 1.112 | 1.122 | 1.123 | 1.132 | 1.130 | 1.139 | 1.134 | 1.145 | 1.138 | 1.159 | 1.146 | 1.174 | 1.152 | 1.190 | 1.157 |
| 1.000 | | 1.056 | | 1.065 | | 1.069 | | 1.072 | | 1.073 | | 1.074 | | 1.077 | | 1.079 | 1.100 | 1.080 |
| 2.000 | | 1.032 | | 1.036 | | 1.037 | | 1.038 | | 1.038 | | 1.038 | | 1.039 | | 1.039 | | 1.040 |
| 10.000 | | 1.007 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 | | 1.008 |
| 20.000 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 | | 1.004 |

$\sigma_n = 6M/dh^2$; $M$ = magnitude of external bending moment; $d$ = width of minimum section; $h$ = plate thickness

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## NOMENCLATURE

| | |
|---|---|
| $a$ | Radius or half-width of minimum section (width of minimum section for flat bar with single notch) |
| $D$ | Cylindrical diameter |
| $d$ | Diameter or width of minimum section |
| $h$ | Plate thickness |
| $K_t$ | Stress concentration factor (SCF) based on minimum section |
| $K_{td}$ | SCF of deep notch |
| $K_{tE}$ | SCF of semi-elliptical notch in semi-infinite plate |
| $K_{tH}$ | SCF of elliptical hole in infinite plate $= 1 + 2\sqrt{t/\rho}$ |
| $K_{tN}$ | SCF of Neuber formula |
| $K_{ts}$ | SCF of shallow notch |
| $K_{tv}$ | SCF of 60° V-shaped notch in semi-infinite plate |
| $M$ | Magnitude of external bending moment |
| $P$ | Magnitude of external load |
| $T$ | Magnitude of external torsional moment |
| $t$ | depth of notch |
| $\epsilon$ | Relative notch radius $= 2\rho/D$ |
| $\nu$ | $= \sqrt{\rho/t}$ |
| $\lambda$ | Relative notch depth $= 2t/D$ |
| $\nu$ | Poisson's ratio (=0.3) |
| $\rho$ | Root radius of notch |
| $\sigma_{max}$ | Maximum stress at root of notch |
| $\sigma_n$ | Nominal stress for minimum section |
| $\xi$ | $= \sqrt{t/\rho}$ |

## APPENDIX

Approximate formulae obtained by applying the least-square method to the exact values of $K_t/K_{tN}$ are expressed as follows.

Problem (a):

$$\begin{aligned} K_t/K_{tN} = {} & (0.9670 + 0.6823\epsilon - 4.5949\epsilon^2) \\ & +(1.3638 - 24.7892\epsilon + 158.378\epsilon^2)\lambda \\ & +(-6.8877 + 211.763\epsilon - 1356.21\epsilon^2)\lambda^2 \\ & +(21.2643 - 810.957\epsilon + 5172.28\epsilon^2)\lambda^3 \\ & +(-38.1300 + 1538.05\epsilon - 9776.43\epsilon^2)\lambda^4 \\ & +(34.8030 - 1406.84\epsilon + 8890.48\epsilon^2)\lambda^5 \\ & +(-12.3798 + 492.074\epsilon - 3083.73\epsilon^2)\lambda^6 \\ & (0.03 \leq \epsilon \leq 0.1,\ 0.02 \leq \lambda \leq 1.0) \end{aligned}$$

$$\begin{aligned} K_t/K_{tN} = {} & (0.9907 - 0.0320\epsilon + 0.0268\epsilon^2) \\ & +(0.3967 + 1.3381\epsilon - 0.8651\epsilon^2)\lambda \\ & +(1.3055 - 12.2708\epsilon + 6.6980\epsilon^2)\lambda^2 \\ & +(-9.2572 + 40.3615\epsilon - 22.8963\epsilon^2)\lambda^3 \\ & +(19.1186 - 73.1167\epsilon + 46.5492\epsilon^2)\lambda^4 \\ & +(-17.9846 + 69.0592\epsilon - 48.0002\epsilon^2)\lambda^5 \\ & +(6.4306 - 25.3391\epsilon + 18.4873\epsilon^2)\lambda^6 \\ & (0.1 < \epsilon \leq 1.0,\ 0.02 \leq \lambda \leq 1.0) \end{aligned} \quad (20)$$

Problem (b):

$$\begin{aligned} K_t/K_{tN} = {} & (0.9859 - 0.2658\epsilon + 1.5531\epsilon^2) \\ & +(1.1593 - 5.2297\epsilon + 25.9976\epsilon^2)\lambda \\ & +(-4.5477 + 35.1032\epsilon - 222.384\epsilon^2)\lambda^2 \\ & +(8.8976 - 126.382\epsilon + 836.366\epsilon^2)\lambda^3 \\ & +(-10.5167 + 245.239\epsilon - 1603.87\epsilon^2)\lambda^4 \\ & +(7.5008 - 239.283\epsilon + 1531.55\epsilon^2)\lambda^5 \\ & +(-2.4786 + 90.8033\epsilon - 569.122\epsilon^2)\lambda^6 \\ & (0.03 \leq \epsilon\ 0.1,\ 0.02 \leq \lambda \leq 1.0) \end{aligned}$$

$$\begin{aligned} K_t/K_{tN} = {} & (0.9738 + 0.0003\epsilon + 0.0148\epsilon^2) \\ & +(0.8799 + 0.2328\epsilon - 0.4536\epsilon^2)\lambda \\ & +(-2.4147 - 9.7679\epsilon + 7.2604\epsilon^2)\lambda^2 \\ & +(0.8960 + 42.2626\epsilon - 28.0362\epsilon^2)\lambda^3 \\ & +(4.6763 - 76.4670\epsilon + 48.9434\epsilon^2)\lambda^4 \\ & +(-6.6692 + 64.4563\epsilon - 40.9527\epsilon^2)\lambda^5 \\ & +(2.6578 - 20.7164\epsilon + 13.2229\epsilon^2)\lambda^6 \\ & (0.1 < \epsilon \leq 1.0,\ 0.02 \leq \lambda \leq 1.0) \end{aligned} \quad (21)$$

Problem (c):

$$\begin{aligned} K_t/K_{tN} = {} & (0.9886 - 0.9685\epsilon + 7.1182\epsilon^2) \\ & +(1.5851 - 7.7911\epsilon - 8.7180\epsilon^2)\lambda \\ & +(-7.6809 + 65.0594\epsilon - 56.9974\epsilon^2)\lambda^2 \\ & +(17.4340 - 179.344\epsilon + 82.7721\epsilon^2)\lambda^3 \\ & +(-21.2671 + 242.037\epsilon + 88.1990\epsilon^2)\lambda^4 \\ & +(13.8080 - 176.775\epsilon - 115.711\epsilon^2)\lambda^5 \\ & +(-3.8676 + 57.7794\epsilon + 3.3695\epsilon^2)\lambda^6 \\ & (0.02 \leq \epsilon \leq 0.1,\ 0.02 \leq \lambda \leq 1.0) \end{aligned}$$

$$\begin{aligned} K_t/K_{tN} = {} & (0.9590 + 0.0460\epsilon - 0.0207\epsilon^2) \\ & +(0.7972 - 1.3067\epsilon + 0.8141\epsilon^2)\lambda \\ & +(-1.6853 + 1.2552\epsilon - 1.2308\epsilon^2)\lambda^2 \\ & +(-0.6959 + 7.8264\epsilon - 2.7312\epsilon^2)\lambda^3 \\ & +(6.0103 - 21.7877\epsilon - 9.5555\epsilon^2)\lambda^4 \\ & +(-6.9476 + 21.2873\epsilon - 9.8461\epsilon^2)\lambda^5 \\ & +(2.5625 - 7.3206\epsilon - 3.4592\epsilon^2)\lambda^6 \\ & (0.1 < \epsilon \leq 1.0,\ 0.02 \leq \lambda \leq 1.0) \end{aligned} \quad (22)$$

Problem (d):

$$\begin{aligned} K_t/K_{tN} = {} & (0.9863 - 0.1967\epsilon - 1.2261\epsilon^2) \\ & +(0.7598 - 14.6944\epsilon + 85.2855\epsilon^2)\lambda \\ & +(-3.3994 + 137.860\epsilon - 770.087\epsilon^2)\lambda^2 \\ & +(11.8465 - 546.326\epsilon + 2964.65\epsilon^2)\lambda^3 \\ & +(-24.6062 + 1074.41\epsilon - 5684.17\epsilon^2)\lambda^4 \\ & +(24.9403 - 1013.22\epsilon + 5212.60\epsilon^2)\lambda^5 \\ & +(-9.5266 + 361.756\epsilon - 1806.89\epsilon^2)\lambda^6 \\ & (0.02 \leqslant \epsilon \leqslant 0.1,\ 0.02 \leqslant \lambda \leqslant 1.0) \end{aligned}$$

$$\begin{aligned} K_t/K_{tN} = {} & (0.9973 - 0.0411\epsilon + 0.0290\epsilon^2) \\ & +(0.0025 + 1.4755\epsilon - 0.9146\epsilon^2)\lambda \\ & +(3.9929 - 13.7614\epsilon + 8.9229\epsilon^2)\lambda^2 \\ & +(-18.6927 + 59.2470\epsilon - 42.7560\epsilon^2)\lambda^3 \\ & +(38.2885 - 130.019\epsilon + 97.5542\epsilon^2)\lambda^4 \\ & +(-36.9392 + 131.133\epsilon - 98.5547\epsilon^2)\lambda^5 \\ & +(13.3503 - 48.0315\epsilon + 35.7172\epsilon^2)\lambda^6 \\ & (0.1 < \epsilon \leqslant 1.0,\ 0.02 \leqslant \lambda \leqslant 1.0) \qquad (23) \end{aligned}$$

Problem (e):

$$\begin{aligned} K_t/K_{tN} = {} & (0.9860 - 0.2543\epsilon + 1.3536\epsilon^2) \\ & +(0.8141 - 1.8146\epsilon + 7.7454\epsilon^2)\lambda \\ & +(-1.9019 - 2.7027\epsilon + 10.6611\epsilon^2)\lambda^2 \\ & +(0.8497 + 37.6317\epsilon - 206.130\epsilon^2)\lambda^3 \\ & +(1.4758 - 70.8874\epsilon + 404.947\epsilon^2)\lambda^4 \\ & +(-1.2098 + 38.0317\epsilon - 220.164\epsilon^2)\lambda^5 \\ & +(-0.0139 - 0.0054\epsilon + 1.5961\epsilon^2)\lambda^6 \\ & (0.02 \leqslant \epsilon \leqslant 0.1,\ 0.02 \leqslant \lambda \leqslant 1.0) \end{aligned}$$

$$\begin{aligned} K_t/K_{tN} = {} & (0.9735 + 0.0036\epsilon + 0.0070\epsilon^2) \\ & +(0.7227 - 0.2412\epsilon + 0.0230\epsilon^2)\lambda \\ & +(-1.8034 - 2.2175\epsilon + 1.3980\epsilon^2)\lambda^2 \\ & +(1.7308 + 7.0844\epsilon - 3.3948\epsilon^2)\lambda^3 \\ & +(-0.7339 - 6.8807\epsilon + 2.6137\epsilon^2)\lambda^4 \\ & +(0.1099 + 2.2525\epsilon - 0.6471\epsilon^2)\lambda^5 \\ & (0.1 < \epsilon \leqslant 1.0,\ 0.02 \leqslant \lambda \leqslant 1.0) \qquad (24) \end{aligned}$$

Problem (f):

$$\begin{aligned} K_t/K_{tN} = {} & (0.9828 - 0.4343\epsilon + 3.5438\epsilon^2) \\ & +(1.5236 - 25.7113\epsilon + 117.095\epsilon^2)\lambda \\ & +(-11.1918 + 292.716\epsilon - 1555.67\epsilon^2)\lambda^2 \\ & +(39.1363 - 1135.95\epsilon + 6288.58\epsilon^2)\lambda^3 \\ & +(-70.1820 + 2101.26\epsilon - 11859.8\epsilon^2)\lambda^4 \\ & +(62.0517 - 1869.87\epsilon + 10639.5\epsilon^2)\lambda^5 \\ & +(-21.3202 + 637.976\epsilon - 3633.18\epsilon^2)\lambda^6 \\ & (0.02 \leqslant \epsilon \leqslant 0.1,\ 0.02 \leqslant \lambda \leqslant 1.0) \end{aligned}$$

$$\begin{aligned} K_t/K_{tN} = {} & (0.9735 + 0.0227\epsilon - 0.0096\epsilon^2) \\ & +(0.0685 - 0.0311\epsilon + 0.2248\epsilon^2)\lambda \\ & +(3.3873 - 5.5445\epsilon + 1.4683\epsilon^2)\lambda^2 \\ & +(-14.7925 + 25.3049\epsilon - 10.1166\epsilon^2)\lambda^3 \\ & +(27.2560 - 51.9612\epsilon + 25.1774\epsilon^2)\lambda^4 \\ & +(-23.9118 + 49.8997\epsilon - 26.5951\epsilon^2)\lambda^5 \\ & +(8.0185 - 17.6893\epsilon + 9.8502\epsilon^2)\lambda^6 \\ & (0.1 < \epsilon \leqslant 1.0,\ 0.02 \leqslant \lambda \leqslant 1.0) \qquad (25) \end{aligned}$$

Problem (g):

$$\begin{aligned} K_t/K_{tN} = {} & (0.9680 + 0.0515\epsilon + 0.2553\epsilon^2) \\ & +(2.3534 - 54.8262\epsilon + 320.423\epsilon^2)\lambda \\ & +(-10.0940 + 354.108\epsilon - 2209.81\epsilon^2)\lambda^2 \\ & +(28.8853 - 1172.25\epsilon + 7508.45\epsilon^2)\lambda^3 \\ & +(-48.6517 + 2057.86\epsilon - 13355.2\epsilon^2)\lambda^4 \\ & +(42.4796 - 1814.50\epsilon - 11838.8\epsilon^2)\lambda^5 \\ & +(-14.9403 + 629.540\epsilon - 4102.83\epsilon^2)\lambda^6 \\ & (0.02 \leqslant \epsilon \leqslant 0.1,\ 0.02 \leqslant \lambda \leqslant 1.0) \end{aligned}$$

$$\begin{aligned} K_t/K_{tN} = {} & (0.9743 + 0.0182\epsilon - 0.0059\epsilon^2) \\ & +(0.0164 + 0.2120\epsilon + 0.0315\epsilon^2)\lambda \\ & +(4.1307 - 8.7720\epsilon + 3.9795\epsilon^2)\lambda^2 \\ & +(-16.4250 + 33.6105\epsilon - 16.8787\epsilon^2)\lambda^3 \\ & +(29.2237 - 63.4641\epsilon + 34.8586\epsilon^2)\lambda^4 \\ & +(-25.7567 + 59.9965\epsilon - 34.9715\epsilon^2)\lambda^5 \\ & +(8.8363 - 21.6003\epsilon + 12.9858\epsilon^2)\lambda^6 \\ & (0.1 < \epsilon \leqslant 1.0,\ 0.02 \leqslant \lambda \leqslant 1.0) \qquad (26) \end{aligned}$$

Problem (h):

$$\begin{aligned} K_t/K_{tN} = {} & (0.9753 - 0.0808\epsilon + 0.8503\epsilon^2) \\ & +(3.2744 - 1.0955\epsilon + 18.3102\epsilon^2)\lambda \\ & +(-25.0147 + 117.504\epsilon - 889.632\epsilon^2)\lambda^2 \\ & +(114.575 - 1090.16\epsilon + 7294.12\epsilon^2)\lambda^3 \\ & +(-360.114 + 4991.87\epsilon - 30783.5\epsilon^2)\lambda^4 \\ & +(742.624 - 12423.4\epsilon + 72901.5\epsilon^2)\lambda^5 \\ & +(-926.385 + 17077.4\epsilon - 97364.7\epsilon^2)\lambda^6 \\ & +(625.865 - 12185.8\epsilon + 68280.4\epsilon^2)\lambda^7 \\ & +(-174.799 + 3513.70\epsilon - 19457.1\epsilon^2)\lambda^8 \\ & (0.02 \leqslant \epsilon \leqslant 0.1,\ 0.02 \leqslant \lambda \leqslant 1.0) \end{aligned}$$

$$\begin{aligned} K_t/K_{tN} = {} & (0.9689 + 0.0615\epsilon - 0.0182\epsilon^2) \\ & +(3.4645 - 0.1304\epsilon - 1.3124\epsilon^2)\lambda \\ & +(-20.4714 - 31.1658\epsilon + 30.3838\epsilon^2)\lambda^2 \\ & +(62.7912 + 244.218\epsilon - 198.562\epsilon^2)\lambda^3 \\ & +(-112.386 - 850.864\epsilon + 649.313\epsilon^2)\lambda^4 \\ & +(122.819 + 1610.82\epsilon - 1196.06\epsilon^2)\lambda^5 \\ & +(-79.5397 - 1728.99\epsilon + 1269.66\epsilon^2)\lambda^6 \\ & +(26.3044 + 992.562\epsilon - 726.854\epsilon^2)\lambda^7 \\ & +(-2.9512 - 236.518\epsilon + 173.457\epsilon^2)\lambda^8 \\ & (0.1 < \epsilon \leqslant 1.0,\ 0.02 \leqslant \lambda \leqslant 1.0) \qquad (27) \end{aligned}$$

Problem (i):

$$\begin{aligned} K_t/K_{tN} = {} & (0.9884 - 0.1496\epsilon + 0.7284\epsilon^2) \\ & +(0.1119 - 8.8807\epsilon + 45.8158\epsilon^2)\lambda \\ & +(6.2418 - 8.9062\epsilon - 0.1946\epsilon^2)\lambda^2 \\ & +(-62.9087 + 490.511\epsilon - 2409.41\epsilon^2)\lambda^3 \\ & +(260.447 - 2737.61\epsilon + 14255.7\epsilon^2)\lambda^4 \\ & +(-581.518 + 7051.88\epsilon - 37105.4\epsilon^2)\lambda^5 \\ & +(728.879 - 9545.72\epsilon + 49936.8\epsilon^2)\lambda^6 \\ & +(-481.611 + 6563.61\epsilon - 33844.0\epsilon^2)\lambda^7 \\ & +(130.370 - 1804.74\epsilon + 9120.10\epsilon^2)\lambda^8 \\ & (0.02 \leqslant \epsilon \leqslant 0.1,\ 0.02 \leqslant \lambda \leqslant 1.0) \end{aligned}$$

$$\begin{aligned} K_t/K_{tN} = {} & (0.9825 - 0.0209\epsilon + 0.0181\epsilon^2) \\ & +(-0.3565 - 0.1201\epsilon - 0.0287\epsilon^2)\lambda \\ & +(6.6071 - 12.9009\epsilon + 7.7073\epsilon^2)\lambda^2 \\ & +(-45.7747 + 90.3018\epsilon - 53.8534\epsilon^2)\lambda^3 \\ & +(151.246 - 288.514\epsilon + 174.893\epsilon^2)\lambda^4 \\ & +(-282.087 + 531.679\epsilon - 330.101\epsilon^2)\lambda^5 \\ & +(305.651 - 580.612\epsilon + 368.422\epsilon^2)\lambda^6 \\ & +(-180.007 + 349.955\epsilon - 225.865\epsilon^2)\lambda^7 \\ & +(44.7389 - 90.0089\epsilon + 58.8076\epsilon^2)\lambda^8 \\ & (0.1 < \epsilon \leqslant 1.0,\ 0.02 \leqslant \lambda \leqslant 1.0) \qquad (28) \end{aligned}$$

Problem (j):

$$\begin{aligned} K_t/K_{tN} = {} & (0.9824 - 0.4700\epsilon - 3.8286\epsilon^2) \\ & +(1.5485 - 24.0474\epsilon + 103.497\epsilon^2)\lambda \\ & +(-11.4932 + 274.566\epsilon - 1407.17\epsilon^2)\lambda^2 \\ & +(40.8929 - 1061.63\epsilon + 5688.36\epsilon^2)\lambda^3 \\ & +(-74.5581 + 1966.49\epsilon - 10782.0\epsilon^2)\lambda^4 \\ & +(66.9231 - 1756.97\epsilon + 9746.39\epsilon^2)\lambda^5 \\ & +(-23.2951 + 602.050\epsilon - 3352.85\epsilon^2)\lambda^6 \\ & (0.02 \leqslant \epsilon \leqslant 0.1,\ 0.02 \leqslant \lambda \leqslant 1.0) \end{aligned}$$

$$
\begin{aligned}
K_t/K_{tN} = {} & (0.9716 + 0.0298\epsilon - 0.0155\epsilon^2) \\
& +(0.1535 - 0.3239\epsilon + 0.4657\epsilon^2)\lambda \\
& +(2.4193 - 1.9976\epsilon - 1.2530\epsilon^2)\lambda^2 \\
& +(-10.1592 + 9.2736\epsilon + 1.8193\epsilon^2)\lambda^3 \\
& +(17.2203 - 18.1439\epsilon + 0.5869\epsilon^2)\lambda^4 \\
& +(-13.8144 + 16.6886\epsilon - 3.0224\epsilon^2)\lambda^5 \\
& +(4.2090 - 5.5266\epsilon + 1.4187\epsilon^2)\lambda^6 \\
& (0.1 < \epsilon \leqslant 1.0,\ 0.02 \leqslant \lambda \leqslant 1.0) \quad (29)
\end{aligned}
$$

Problem (k):

$$
\begin{aligned}
K_t/K_{tN} = {} & (0.9731 - 0.1654\epsilon + 1.7774\epsilon^2) \\
& +(2.1131 - 44.4614\epsilon + 246.835\epsilon^2)\lambda \\
& +(-7.2436 + 231.854\epsilon - 1334.42\epsilon^2)\lambda^2 \\
& +(15.9011 - 608.022\epsilon + 3459.76\epsilon^2)\lambda^3 \\
& +(-21.5831 + 870.224\epsilon - 4828.75\epsilon^2)\lambda^4 \\
& +(16.6009 - 665.511\epsilon + 3593.54\epsilon^2)\lambda^5 \\
& +(-5.7622 + 216.102\epsilon - 1138.89\epsilon^2)\lambda^6 \\
& (0.02 \leqslant \epsilon \leqslant 0.1,\ 0.02 \leqslant \lambda \leqslant 1.0)
\end{aligned}
$$

$$
\begin{aligned}
K_t/K_{tN} = {} & (0.9721 + 0.0274\epsilon - 0.0134\epsilon^2) \\
& +(0.1173 - 0.1578\epsilon + 0.3336\epsilon^2)\lambda \\
& +(3.0372 - 4.6498\epsilon + 0.8133\epsilon^2)\lambda^2 \\
& +(-11.4229 + 16.0192\epsilon - 3.7807\epsilon^2)\lambda^3 \\
& +(18.6417 - 27.6880\epsilon + 8.9209\epsilon^2)\lambda^4 \\
& +(-15.2484 + 25.6479\epsilon - 10.7340\epsilon^2)\lambda^5 \\
& +(4.9031 - 9.1986\epsilon + 4.4601\epsilon^2)\lambda^6 \\
& (0.1 < \epsilon \leqslant 1.0,\ 0.02 \leqslant \lambda \leqslant 1.0) \quad (30)
\end{aligned}
$$