Int. J. Fatigue Vol. 19, No. 1, pp. 75–84, 1997

0142-1123/97/$17.00+.00

PII: S0142-1123(96)00045-X

Stress concentration factors for shoulder fillets in round and flat bars under various loads

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(Received 13 November 1995)

The stress concentration problem of shoulder fillets in round and flat bars under various loads is often encountered in machine design of shafts. Also it is important for test specimens used to investigate the mechanical properties of materials. Accurate stress concentration factors (SCFs) have been given in a recent analysis of the body force method. However, the results of the solutions have been presented in tabular form which is not suitable for engineering applications. For notched bars, Neuber proposed the simple approximate formula K_{tN} which is useful for a wide range of notch shape: $1/(K_{tN} - 1)^m = 1/(K_{ts} - 1)^m + 1/(K_{td} - 1)^m$ and $m = 2$. Here, K_{ts} and K_{td} are exact solutions for shallow and deep notches, respectively. Neuber's simple formula has been used for >40 years in the design of notched bars because of its convenience. In this study, similar convenient equations K_{tN} are initially proposed as an extension of Neuber's formula to the problem of shoulder fillet. In this formula new definitions of K_{ts} and K_{td} are used corresponding to two extreme cases of shoulder fillet in round and flat bars. Next, the most suitable exponent m is determined so as to minimize the difference between K_{tN} and accurate K_t, that is, the results of the body force method. Next, by applying the least squares method to the ratio K_t/K_{tN} more accurate formulas are proposed. The formulas proposed in this paper are found to give the stress concentration factors with better than 1% accuracy. In addition, the stress concentration factors are also provided in a graphical way on the basis of the formula so they can be used easily in design or research. Copyright © 1997 Elsevier Science Limited.

(Keywords: stress concentration factor; shoulder fillet; numerical analysis; round bar; flat bar; tension; bending; torsion)

The stress concentration problem of shoulder fillet in round and flat bars, as shown in *Figure 1* is often encountered in machine design of shaft. Also it is important for the test specimens used in order to investigate the fatigue strength of materials. Accurate stress concentration factors (SCFs) have been given in a recent analysis of the body force method.[1–6] The results have shown that Peterson's stress concentration factors[7] have a non-conservative error of about 10% for a wide geometrical range of fillets. However, accurate SCFs have not been given in the form of formulas suitable for engineering applications.

On the other hand, in the problem of notched bars, Neuber proposed a simple ingenious approximate formula K_{tN} useful for a wide range of notch shapes:[8]

$$\frac{1}{(K_{tN} - 1)^m} = \frac{1}{(K_{ts} - 1)^m} + \frac{1}{(K_{td} - 1)^m} \text{ and } m = 2 \qquad (1)$$

Here, K_{ts} and K_{td} are the exact solutions for shallow and deep notches, respectively. In the preceding paper,[9] for the problem of notch, as a result of comparison between Neuber's results and the results of body force method, correction factors for the Neuber equation are given in the form of a formula by applying the least squares method. In addition, the stress concentration factors are also provided in a graphical way on the basis of the formulas so they can be easily used in

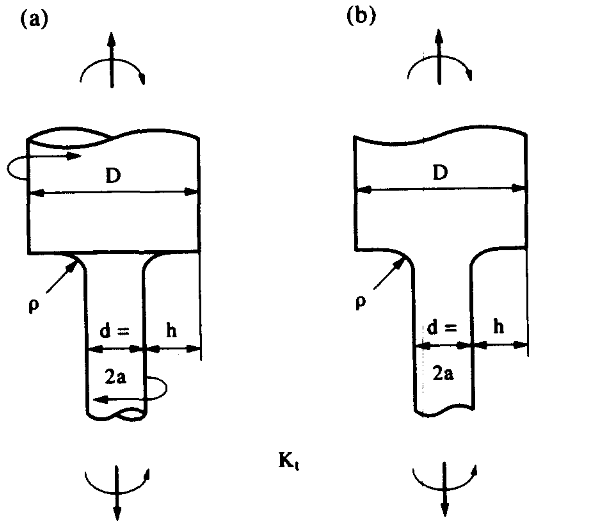

Figure 1 Round and flat bars with shoulder fillets. (a) tension, (b) bending, (c) torsion, (d) tension, (e) bending

design or research. However, neither Neuber's nor other convenient formulas have been given for shoulder fillet in previous research.

In this paper, first, the stress concentration of round and flat bars with fillet is systematically analyzed by the body force method. Second, for the problem of shoulder fillet, similar equations are also proposed as an extension of Neuber's formula. Third, the most suitable exponent m is determined so as to minimize the difference between K_{tN} and accurate K_t, that is, the results of the body force method. Finally, by applying the least squares method to the ratio K_t/K_{tN} more accurate formulas are proposed. In addition, the stress concentration factors are also provided in a graphical way on the basis of the formula so they can be used easily in design or research.

DEFINITION OF STRESS CONCENTRATION FACTORS

In this paper, the SCFs are based on the nominal stress at the minimum diameter or width and defined in Equation (2).

$$K_t = \sigma_{max}/\sigma_n \tag{2}$$

where σ_{max} is the maximum stress at the root of fillet. The problems treated in this paper are shown as follows with the definition of nominal net stress σ_n for each problem (see *Figure 1*):

Problem (a): fillet in round bar under tension [$\sigma_n = 4P/(\pi d^2)$]
Problem (b): fillet in round bar under bending [$\sigma_n = 32M/(\pi d^3)$]
Problem (c): fillet in round bar under torsion [$\tau_n = 16T/(\pi d^3)$]
Problem (d): fillet in flat bar under tension [$\sigma_n = P/dt$]
Problem (e): fillet in flat bar under bending [$\sigma_n = 6M/d^2t$]

where: d is a diameter or width of minimum section; t is a plate thickness; P is the magnitude of external load; M is the magnitude of external bending moment; and T is the magnitude of external tortional moment. In the problems of (a), (b), Poisson's ratio ν is assumed to be 0.3. In this study the following notations will be used.

$$\xi = \sqrt{h/\rho},\ \eta = \sqrt{\rho/h},\ \lambda = 2h/D,\ \epsilon = 2\rho/D \tag{3}$$

where the parameters ρ, h, D, d are indicated in *Figure 1*.

PROPOSAL OF AN APPROXIMATE FORMULA FOR THE FILLET AS AN EXTENSION OF NEUBER'S TRIGONOMETRIC RULE

To estimate SCFs of notched bars, the Neuber method makes use of the two exact solutions,[8] that is, the solution of elliptical hole in an infinite plate K_{tH} as a shallow notch K_{ts} and the solution of hyperbolic notch as a deep notch K_{td}. From these values, the Neuber value K_{tN} is given by the following ingenious simple equation:

$$K_{tN} = \frac{(K_{ts}-1)(K_{td}-1)}{\{(K_{ts}-1)^m + (K_{td}-1)^m\}^{1/m}} + 1 \tag{4}$$

In this paper, in order to apply the Neuber formula to fillet problems, SCFs of a shoulder fillet in a semi-infinite plate as shown in *Figure 2(a)* will be used as a solution of K_{ts}.

Table 1 shows SCFs of a shoulder fillet in a semi-infinite plate K_{tF} for various values of h/ρ obtained by the body force method.[2,3] Accurate formulas can be given by applying the least squares method to the ratio K_{tF}/K_{tH} because the variation of K_{tF}/K_{tH} is very small for the whole variation of h/ρ as shown in *Figures 3* and *4*. Here $K_{tH} = 1 + \sqrt{h/\rho}$ means the SCF of an elliptical hole in an infinite plate under uniaxial tension σ when the lower rim of the hole is also subjected to traction σ as shown in *Figure 2(c)*. The reason why K_{tF} is nearly equal to K_{tH} is illustrated in *Figure 2*. As shown in *Figure 2*, first K_{tF} can be approximated by SCF of long rectangular hole with rounded corner in an infinite plate shown in *Figure 2(b)* because the disappearance of normal stress σ_x along the edge of semi-infinite plate does not cause much difference at the root of fillet. Next, the infinitely long rectangular hole (b) is replaced by a finite hole as shown in *Figure 2(c)* and *(d)*. At this replacement, the effect of traction free along the lower rim of long rectangular hole should be neglected and therefore traction σ should be added along the lower rim of finite hole as shown in *Figure 2(c)* and *(d)*. As a result, K_{tF} may be approximated by $K_{tH} = 1 + \sqrt{h/\rho}$ instead of the SCF of traction free elliptical hole $(1 + 2\sqrt{h/\rho})$. The expression of K_{tF} is shown in Equation (5) with <0.2% estimated errors.

(1) $0 \le \xi < 1.0$

$$K_{tF} = (1.000 + 0.159\xi - 0.127\xi^2 + 0.050\xi^3)K_{tH} \tag{5a}$$

(2) $0 < \eta \le 1.0\ (1.0 \le \xi < \infty)$

$$K_{tF} = (1.106 + 0.016\eta - 0.059\eta^2 + 0.019\eta^3)K_{tH} \tag{(5b)}$$

On the other hand, as a solution of K_{td}, it is desirable to use a deep shoulder fillet as shown in *Figure 5*. However, the solution cannot be given in a convenient form. In this paper, therefore, SCFs of a deep hyperbolic notch[8] will be applied for practical use. They are expressed in Equations (6)–(10). In *Table 2*, SCFs of deep hyperbolic notches and the results of body force method are compared. As shown in *Table 2*, they are in good agreement in the range $2h/D > 0.6$. Finally, in each loading condition, K_{ts} and K_{td} are expressed as follows.

Problem (a):

$$K_{ts} = K_{tF}$$

$$K_{td} = \frac{1}{N}\left\{\frac{a}{\rho}\sqrt{\frac{a}{\rho}+1} + (0.5+\nu)\frac{a}{\rho} + (1+\nu)\left(\sqrt{\frac{a}{\rho}+1}+1\right)\right\}$$

$$N = \frac{a}{\rho} + 2\nu\sqrt{\frac{a}{\rho}+1} + 2 \tag{6}$$

Problem (b):

$$K_{ts} = K_{tF}$$

$$K_{td} = \frac{1}{N}\frac{3}{4}\left(\sqrt{\frac{a}{\rho}+1}+1\right)\left\{3\frac{a}{\rho}\right.$$

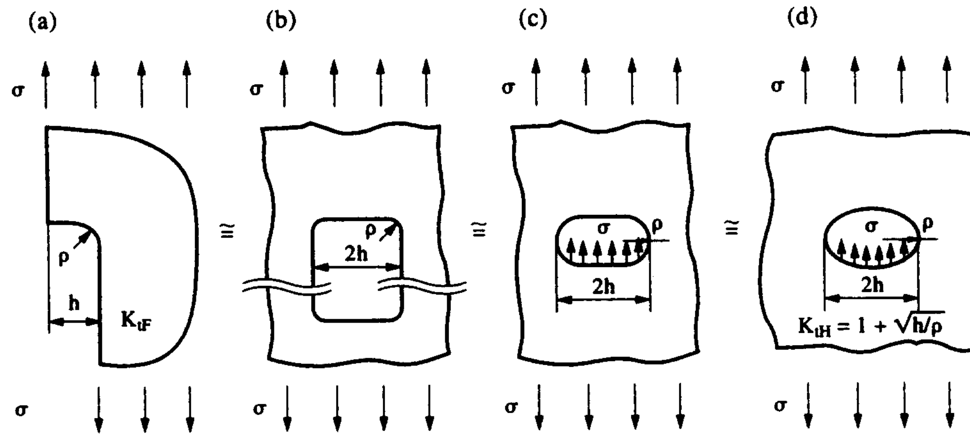

Figure 2 SCF of a fillet in a semi-infinite plate K_{tF} can be approximated by $K_{tH} = 1 + \sqrt{h/\rho}$

Table 1 K_t of a semi-infinite plate with a fillet

h/ρ	ρ/h	K_{tF}	K_{tH}	K_{tF}/K_{tH}	Equation (5)
0.0000				1.0000	1.0000
0.0625	16.00	1.291	1.2500	1.0328	1.0326
0.1250	8.00	1.412	1.3536	1.0431	1.0425
0.2500	4.00	1.581	1.5000	1.0540	1.0540
0.3333	3.00	1.669	1.5774	1.0581	1.0591
0.5000	2.00	1.824	1.7071	1.0685	1.0666
0.6666	1.50	1.946	1.8165	1.0713	1.0724
0.8000	1.25	2.041	1.8944	1.0774	1.0764
1.0000	1.00	2.164	2.0000	1.0820	1.0820
1.00	1.0000	2.164	2.0000	1.0820	1.0820
1.25	0.8000	2.300	2.1180	1.0859	1.0867
1.50	0.6666	2.423	2.2247	1.0891	1.0901
2.00	0.5000	2.640	2.4142	1.0935	1.0945
3.00	0.3333	3.007	2.7321	1.1006	1.0992
4.00	0.2500	3.304	3.0000	1.1013	1.1016
8.00	0.1250	4.230	3.8284	1.1049	1.1051
16.00	0.0625	5.527	5.0000	1.1054	1.1066
	0.0000			1.1059	1.1060

$$-(1-2\nu)\sqrt{\frac{a}{2\rho}+1}+4+\nu\Big\}$$

$$N = 3\left(\frac{a}{\rho}+1\right)+(1+4\nu)$$

$$\sqrt{\frac{a}{\rho}+1}+(1+\nu)\Big/\left(1+\sqrt{\frac{a}{\rho}+1}\right) \qquad (7)$$

Problem (c):

$$K_{ts} = 1 + \frac{1}{2}\sqrt{\frac{t}{\rho}}$$

$$K_{td} = \frac{3(1+\sqrt{a/\rho+1})^2}{4\left(1+2\sqrt{a/\rho+1}\right)} \qquad (8)$$

Problem (d):

$$K_{ts} = K_{tF}$$

$$K_{td} = \frac{2(a/\rho+1)\sqrt{a/\rho}}{(a/\rho+1)\tan^{-1}\sqrt{a/\rho}+\sqrt{a/\rho}} \qquad (9)$$

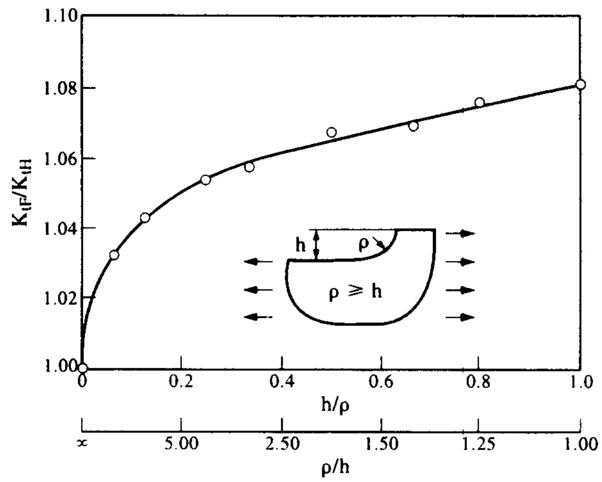

Figure 3 K_t of a semi-infinite plate with a fillet ($K_{tH} = 1 + \sqrt{h/\rho}$)

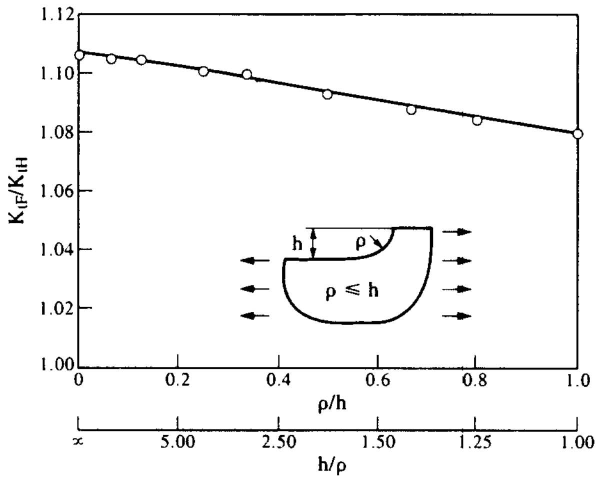

Figure 4 K_t of a semi-infinite plate with a fillet ($K_{tH} = 1 + \sqrt{h/\rho}$)

Problem (e):

$$K_{ts} = K_{tF}$$

$$K_{td} = \frac{4a/\rho \times \sqrt{a/\rho}}{3\{\sqrt{a/\rho}+(a/\rho-1)\tan^{-1}\sqrt{a/\rho}\}} \qquad (10)$$

In *Tables 3–7*, the proposed new Neuber formulas (4)

Table 2 SCF of deep shoulder fillets in bars K_t is approximated by SCF of deep shoulder fillets K_{td}

$2h/D$	Round bar under tension				Round bar under bending				Round bar under torsion				Flat bar under tension				Flat bar under bending			
	$2\rho/D = 0.5$		$2\rho/D = 1.0$		$2\rho/D = 0.5$		$2\rho/D = 1.0$		$2\rho/D = 0.5$		$2\rho/D = 1.0$		$2\rho/D = 0.5$		$2\rho/D = 1.0$		$2\rho/D = 0.5$		$2\rho/D = 1.0$	
	K_t	K_{td}	K_t	K_{td}	K_t	K_{td}	K_t	K_{td}	K_t	K_{td}	K_t	K_{td}	K_t	K_{td}	K_t	K_{td}	K_t	K_{td}	K_t	K_{td}
0.50	1.346	1.391	1.175	1.207	1.227	1.264	1.125	1.139	1.104	1.142	1.054	1.076	1.515	1.556	1.286	1.301	1.286	1.333	1.148	1.180
0.60	1.276	1.320	1.138	1.168	1.178	1.215	1.105	1.113	1.084	1.117	1.043	1.062	1.439	1.458	1.227	1.245	1.229	1.275	1.121	1.147
0.70	1.204	1.245	1.101	1.127	1.131	1.165	1.082	1.085	1.062	1.090	1.029	1.047	1.333	1.355	1.165	1.188	1.175	1.213	1.091	1.112
0.80	1.134	1.168	1.070	1.086	1.101	1.113	1.057	1.058	1.036	1.062	1.020	1.032	1.215	1.245	1.098	1.128	1.119	1.147	1.062	1.077
0.90	1.069	1.086	1.036	1.044	1.053	1.058	1.026	1.029	1.014	1.032	1.008	1.016	1.096	1.128	1.033	1.065	1.062	1.078	1.034	1.039
1.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

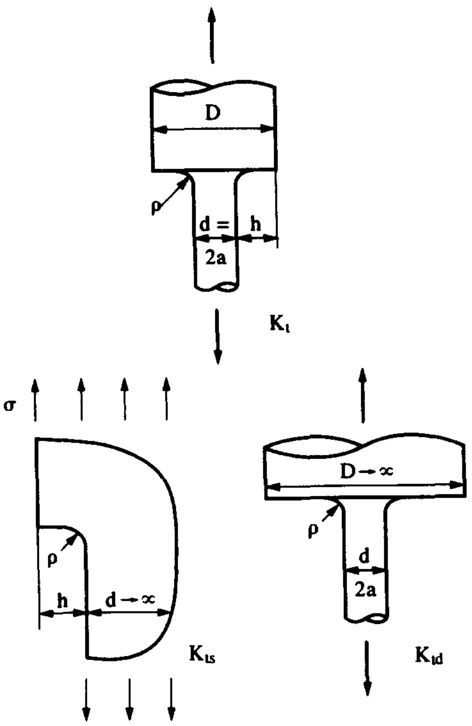

Figure 5 K_{ts} and K_t with a fillet

for shoulder fillet in round and flat bars with exponent $m = 2$ are shown for the wide geometry of fillet in comparison with the accurate stress concentration factors obtained by using the body force method. The analysis method is outlined in Appendix I. As shown in these tables, the formulas with $m = 2$ are found to yield stress concentration factors for a fillet within a 10% error.

CORRECTION OF PROPOSED FORMULA

For notched bars, Neuber proposed Equation (4) with the exponent $m = 2$. However, in this study, the most suitable exponent m is considered so as to minimize the difference between K_{tN} and accurate K_t, which is the result of the body force method. As an example, in the case of flat bar under bending, K_t/K_{tN} values are plotted in *Figure 6* when $m = 2$ and also in *Figure 7* when $m = 1.4$. In *Figure 7* maximum error is <6%, and the accuracy is improved compared with *Figure 6*. In a similar way, the most suitable exponent m is obtained in each problem as shown in Equation (11).

$$
\begin{aligned}
&\text{Problem (a): } m = 1.8 \text{ (error is <5\%)}\\
&\text{Problem (b): } m = 1.6 \text{ (error is <5\%)}\\
&\text{Problem (c): } m = 1.8 \text{ (error is <7\%)}\\
&\text{Problem (d): } m = 1.6 \text{ (error is <4\%)}
\end{aligned}
\tag{11}
$$

Table 3 K_t and K_{tN} with $m = 2$ of a fillet in round bar under tension [$\sigma_n = 4P/(\pi d^2)$, P = magnitude of external load, d = diameter of minimum section]

$2\rho/D$	0.03		0.05		0.1		0.2		0.5		1.0	
$2h/D$	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}
0.05	2.408	2.407	2.078	2.085	1.751	1.756	1.517	1.524	1.312	1.314	1.208	1.206
0.1	2.899	2.870	2.444	2.439	1.992	1.997	1.674	1.677	1.394	1.391	1.252	1.246
0.2	3.398	3.357	2.826	2.798	2.240	2.226	1.816	1.813	1.451	1.443	1.271	1.263
0.3	3.613	3.576	2.975	2.946	2.333	2.304	1.854	1.843	1.446	1.439	1.250	1.252
0.4	3.647	3.634	2.987	2.969	2.315	2.295	1.824	1.817	1.405	1.409	1.214	1.228
0.5	3.548	3.568	2.894	2.898	2.230	2.222	1.748	1.752	1.346	1.363	1.175	1.198
0.6	3.333	3.391	2.714	2.743	2.087	2.096	1.638	1.655	1.276	1.306	1.138	1.163
0.7	3.011	3.100	2.455	2.504	1.895	1.918	1.497	1.532	1.204	1.239	1.101	1.126
0.8	2.579	2.671	2.110	2.167	1.643	1.685	1.340	1.382	1.134	1.166	1.070	1.086
0.9	1.971	2.044	1.649	1.697	1.344	1.387	1.169	1.206	1.069	1.086	1.036	1.044

Table 4 K_t and K_{tN} with $m = 2$ of a fillet in round bar under bending [$\sigma_n = 32M/(\pi d^3)$, M = magnitude of external bending movement, d = diameter of minimum section]

$2\rho/D$	0.03		0.05		0.1		0.2		0.5		1.0	
$2h/D$	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}
0.05	2.329	2.294	2.008	1.989	1.691	1.680	1.469	1.461	1.264	1.264	1.177	1.166
0.1	2.739	2.645	2.304	2.252	1.875	1.849	1.576	1.562	1.323	1.309	1.193	1.186
0.2	3.091	2.953	2.549	2.468	2.034	1.977	1.644	1.628	1.335	1.326	1.189	1.188
0.3	3.163	3.046	2.576	2.521	2.067	1.993	1.632	1.624	1.307	1.313	1.168	1.175
0.4	3.081	3.023	2.494	2.488	2.001	1.955	1.579	1.587	1.270	1.285	1.146	1.156
0.5	2.902	2.918	2.352	2.396	1.891	1.879	1.501	1.528	1.227	1.249	1.125	1.134
0.6	2.667	2.743	2.182	2.252	1.739	1.772	1.418	1.453	1.178	1.208	1.105	1.110
0.7	2.390	2.497	1.981	2.058	1.579	1.636	1.324	1.363	1.131	1.162	1.082	1.084
0.8	2.059	2.168	1.729	1.806	1.411	1.468	1.225	1.259	1.101	1.111	1.057	1.057
0.9	1.624	1.715	1.431	1.474	1.229	1.261	1.122	1.138	1.053	1.057	1.026	1.029

Table 5 K_t and K_{tN} with m = 2 of a fillet in round bar under torsion [$\tau_n = 16T/(\pi d^3)$, T = magnitude of external torsional moment, d = diameter of minimum section]

$2\rho/D$	0.03		0.05		0.1		0.2		0.5		1.0	
$2h/D$	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}
0.05	1.663	1.591	1.498	1.454	1.334	1.315	1.221	1.217	1.124	1.128	1.076	1.083
0.1	1.858	1.776	1.638	1.592	1.420	1.406	1.270	1.273	1.144	1.155	1.085	1.096
0.2	2.011	1.956	1.746	1.721	1.479	1.483	1.297	1.316	1.149	1.170	1.083	1.100
0.3	2.025	2.021	1.753	1.762	1.476	1.502	1.288	1.320	1.138	1.165	1.075	1.095
0.4	1.977	2.022	1.711	1.755	1.443	1.489	1.262	1.305	1.122	1.152	1.065	1.085
0.5	1.893	1.976	1.642	1.714	1.393	1.454	1.229	1.277	1.104	1.134	1.054	1.074
0.6	1.782	1.892	1.555	1.645	1.334	1.402	1.190	1.240	1.084	1.113	1.043	1.061
0.7	1.649	1.770	1.453	1.548	1.266	1.333	1.148	1.194	1.062	1.088	1.029	1.047
0.8	1.489	1.603	1.333	1.420	1.189	1.248	1.101	1.139	1.036	1.061	1.020	1.032
0.9	1.287	1.374	1.186	1.251	1.097	1.141	1.044	1.076	1.014	1.032	1.008	1.016

Table 6 K_t and K_{tN} with m = 2 of fillets in flat bar under tension [$\sigma_n = P/dt$, P = magnitude of external load, d = width of minimum section, t = plate thickness]

$2\rho/D$	0.03		0.05		0.1		0.2		0.5		1.0	
$2h/D$	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}
0.05	2.436	2.411	2.105	2.098	1.776	1.762	1.540	1.531	1.331	1.325	1.224	1.219
0.1	2.955	2.893	2.501	2.462	2.049	2.020	1.726	1.702	1.438	1.419	1.291	1.274
0.2	3.524	3.439	2.953	2.873	2.352	2.295	1.927	1.878	1.541	1.503	1.347	1.315
0.3	3.830	3.728	3.171	3.080	2.499	2.421	2.014	1.948	1.576	1.524	1.352	1.318
0.4	3.957	3.859	3.257	3.163	2.543	2.458	2.027	1.954	1.565	1.510	1.330	1.300
0.5	3.950	3.863	3.233	3.147	2.505	2.424	1.984	1.912	1.515	1.470	1.286	1.269
0.6	3.810	3.745	3.106	3.036	2.397	2.323	1.888	1.826	1.439	1.410	1.227	1.228
0.7	3.533	3.492	2.873	2.822	2.211	2.153	1.739	1.698	1.333	1.332	1.165	1.180
0.8	3.087	3.066	2.511	2.476	1.933	1.901	1.531	1.523	1.215	1.237	1.098	1.125
0.9	2.373	2.366	1.945	1.937	1.528	1.538	1.277	1.295	1.096	1.126	1.033	1.065

Table 7 K_t and K_{tN} with m = 2 of fillets in flat bar under bending [$\sigma_n = 6M/d^2t$), M = magnitude of external bending moment, d = width of minimum section, t = plate thickness]

$2\rho/D$	0.03		0.05		0.1		0.2		0.5		1.0	
$2h/D$	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}	K_t	K_{tN}
0.05	2.348	2.272	2.032	1.974	1.711	1.672	1.484	1.459	1.302	1.270	1.190	1.174
0.1	2.752	2.624	2.335	2.241	1.911	1.849	1.610	1.569	1.369	1.324	1.217	1.203
0.2	3.110	2.957	2.595	2.481	2.072	1.999	1.718	1.657	1.407	1.357	1.221	1.215
0.3	3.202	3.082	2.651	2.562	2.091	2.039	1.703	1.670	1.389	1.353	1.202	1.207
0.4	3.152	3.092	2.599	2.556	2.040	2.020	1.650	1.647	1.344	1.331	1.177	1.189
0.5	3.009	3.018	2.477	2.487	1.943	1.959	1.578	1.597	1.286	1.297	1.148	1.166
0.6	2.797	2.868	2.305	2.362	1.817	1.863	1.490	1.525	1.229	1.254	1.121	1.139
0.7	2.524	2.640	2.091	2.179	1.665	1.730	1.390	1.433	1.175	1.202	1.091	1.109
0.8	2.181	2.314	1.828	1.926	1.489	1.555	1.283	1.318	1.119	1.143	1.062	1.075
0.9	1.727	1.839	1.500	1.570	1.293	1.325	1.162	1.177	1.062	1.058	1.034	1.039

Problem (e): $m = 1.4$ (error is <6%)

Furthermore, the approximate formulas for K_t/K_{tN} for each problems can be obtained by applying the least squares method. These obtained formulas, namely correction factors, are shown in Appendix II. As an example, the curves given by the obtained formulas are shown as the solid lines in *Figure 7*. *Figure 7* indicates that the proposed formulas yield SCFs with better than 1% accuracy.

Stress concentration factors for problems (a)–(e) are provided directly in a graphical way as shown in *Figures 8–12* so they can be easily used in design or research. These figures are obtained from the correction factors in Appendix II with Equations (4)–(11).

CONCLUSION

The stress concentration of shoulder fillets in round and flat bars under various loading is often encountered in the mechanical design of shafts. It is also important for test specimens used to investigate the fatigue strength of materials. In this paper, approximate formulas that are suitable for engineering applications are

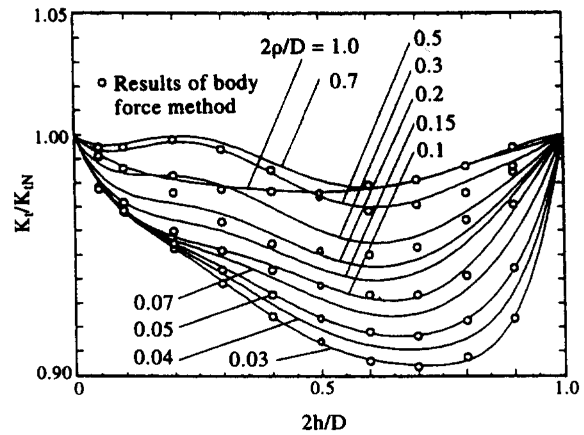

Figure 6 K_t/K_{tN} when $m = 2$ [$\sigma_n = 6M/d^2t$, P = magnitude of external load, d = diameter of minimum section]

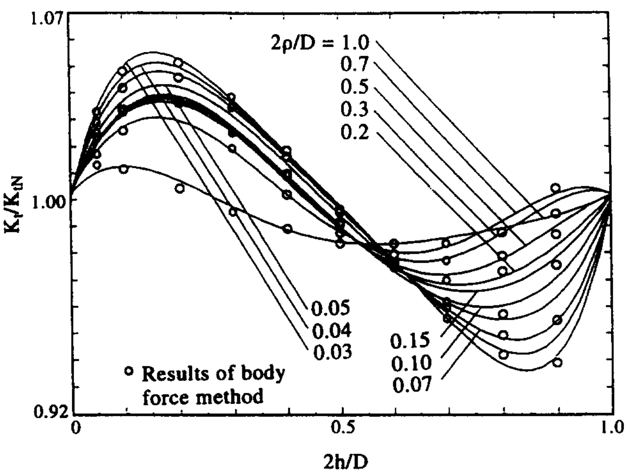

Figure 7 K_t/K_{tN} when $m = 1.4$ [$\sigma_n = 6M/d^2t$, P = magnitude of external load, d = diameter of minimum section]

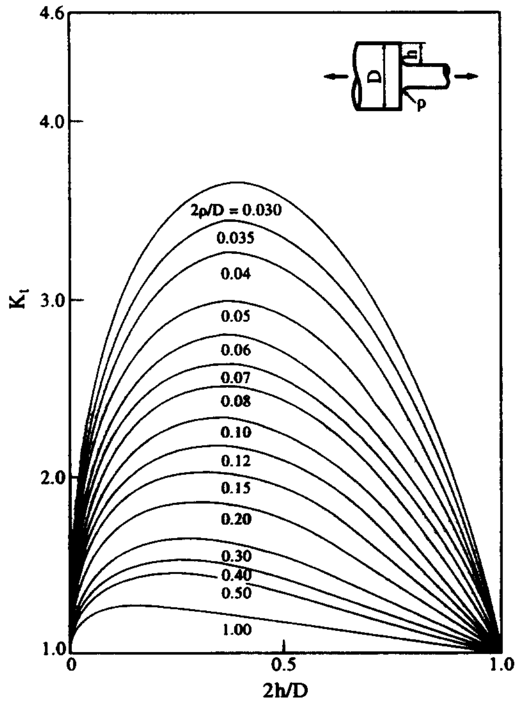

Figure 8 K_t of a fillet in round bar under tension [$\sigma_n = 4P/(\pi d^2)$, P = magnitude of external load, d = diameter of minimum section]

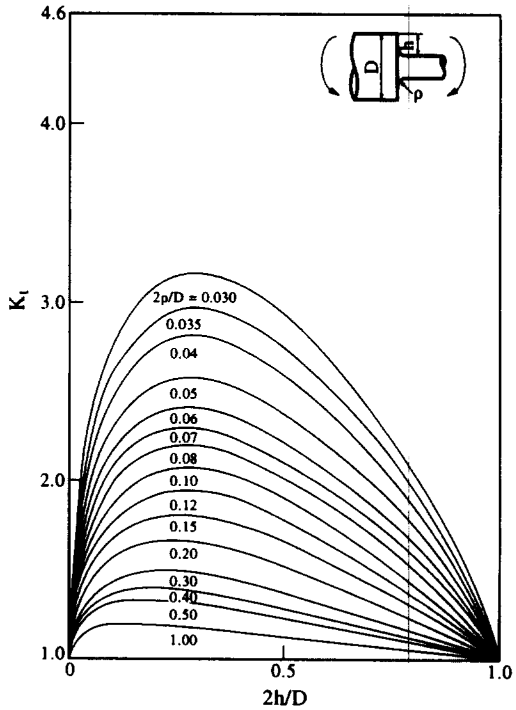

Figure 9 K_t of a fillet in round bar under bending [$\sigma_n = 32M/(\pi d^3)$, M = magnitude of external bending moment, d = diameter of minimum section]

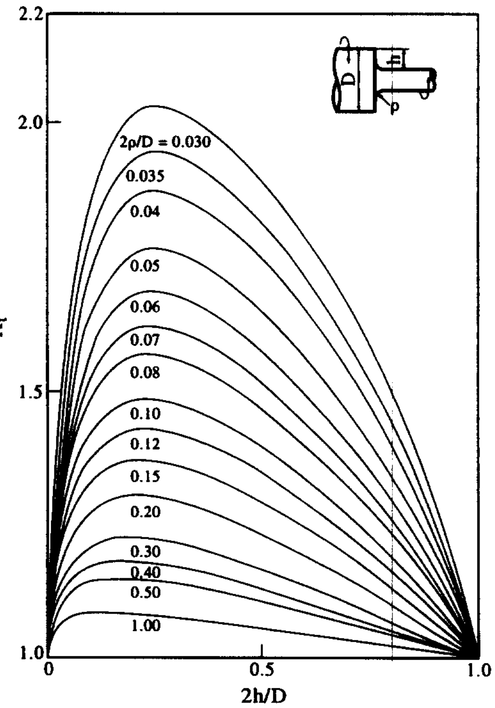

Figure 10 K_t of a fillet in round bar under torsion [$\tau_n = 16T/(\pi d^3)$, T = magnitude of external torsional moment, d = diameter of minimum section]

proposed using the results of the body force method. The conclusions can be made as follows.

1. Accurate SCFs of shoulder fillets in round bar under tension, bending, and torsion and also in flat bars under tension and bending are systematically analyzed using the body force method. They are shown

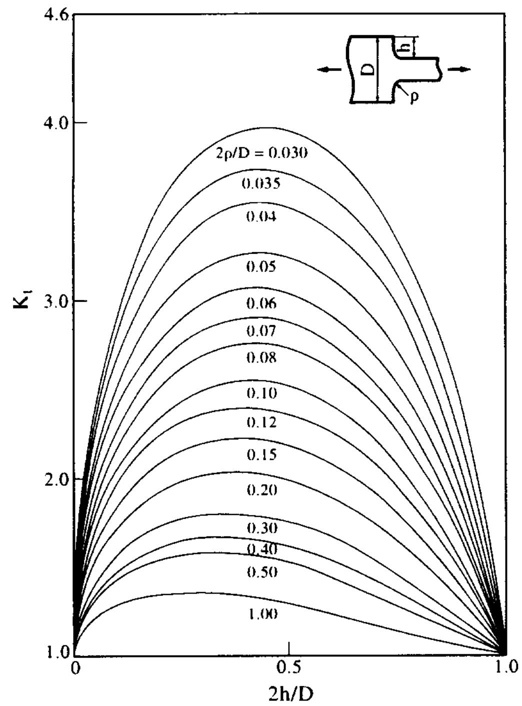

Figure 11 K_t of fillets in flat bar under tension [$\sigma_n = P/dt$, P = magnitude of external load, d = width of minimum section, t = plate thickness]

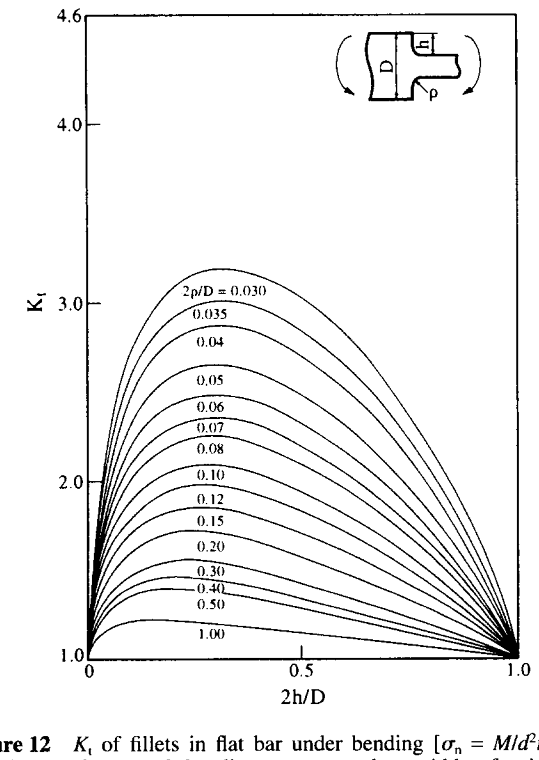

Figure 12 K_t of fillets in flat bar under bending [$\sigma_n = M/d^2t$, M = magnitude of external bending moment, d = width of minimum section, t = plate thickness]

in *Tables 3–7* for the wide geometrical variation of the shoulder fillet.

2. A very accurate formula for SCFs of the shoulder fillet in a semi-infinite plate K_{tF}, which is an extreme case of the shoulder fillet in the bar, was obtained for the whole variation of geometry of the fillet by applying the least squares method to the ratio of the results of the body force method and $K_{tH} = 1 + \sqrt{h/\rho}$ (see *Figures 3* and *4*).
3. Convenient formulas useful for estimating SCFs of shoulder fillets in round and flat bars are proposed as an extension of Neuber's formula, which was originally proposed for notched bars. In this new formula (K_{tN}), SCFs of shoulder fillet in a semi-infinite plate K_{tF}, and SCFs of deep hyperbolic notch are used as two extreme cases of shoulder fillet in bars.
4. The most suitable exponent m of K_{tN} is determined so as to minimize the difference between K_{tN} and accurate K_t, that is, the results of body force method. Finally, by applying the least squares method to the ratio K_t/K_{tN} more accurate formulas are proposed. SCFs are also provided in a graphical way on the basis of the formulas so they can be used easily in design or research.

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NOMENCLATURE

ρ	Root radius of fillet
h	Depth of fillet
d	Diameter or width of minimum section
a	Radius or half-width of minimum section
D	Cyclindrical diameter or width of maximum section
t	Plate thickness
ξ	$= \sqrt{h/\rho}$
η	$= \sqrt{\rho/h}$
λ	Relative fillet depth, $= 2h/D$
ϵ	Relative fillet radius, $= 2\rho/D$
ν	Poisson's ratio (= 0.3)
P	The magnitude of external load
M	The magnitude of external bending moment
T	The magnitude of external torsional moment
σ_{max}	Maximum stress at the root of fillet
σ_n	Nominal stress for the minimum section
K_t	Stress concentration factor (SCF) based on the minimum section
K_{tF}	SCF of a fillet in a semi-infinite plate

K_{tH}	SCF of an elliptical hole in an infinite plate as shown in *Figure 2(d)* $= 1 + \sqrt{h/\rho}$
K_{ts}	SCF of a shallow shoulder fillet
K_{td}	SCF of a deep shoulder fillet
K_{tN}	SCF of the Neuber formula

APPENDIX I: ANALYSIS OF FILLET USING THE BODY FORCE METHOD

The analysis of fillet is more difficult than the ordinary analysis of notch and hole mainly because the position of maximum stress is slightly changed depending on the fillet geometry. In the previous papers,[1–6] some numerical results of stress concentration factors of fillets were shown. In this study, additional and systematic calculations are performed to propose convenient formulas useful for various geometry of fillet under various loads as shown in *Figure 1*. The brief explanation of analysis method is as follows.[1–6,12,13]

Figure A1 illustrates the analysis method by taking examples of problems (a) and (d) in *Figure 1*. For round bars, the stress fields induced by several types of ring forces[10] are used as fundamental solutions; then, the prospective boundaries for the fillet and others are imagined in an infinite body as shown in *Figure A1(a)*. On the other hand, for flat bars, two kinds of semi-infinite plate, whose edges are corresponding to stress free edges of wide part of flat bars, are considered.[12,13] As shown in these figures, body forces are distributed along the prospective boundaries so as to satisfy the boundary conditions. Then, the imaginary boundaries are divided into several intervals and the densities of the body forces, which are assumed to be constant in each interval, are determined from the boundary condition. It should be noted that a comparatively large division number is set around the fillet because the position of maximum stress cannot be known beforehand. The final results of K_t shown in *Tables 3–7* are obtained by the extrapolation from the analysis with the finite division number N because the error due to the finiteness is nearly proportional to $1/N$.

APPENDIX II: CORRECTION FACTORS OF THE FORMULAS: K_t/K_{tN}

Approximate formulas obtained by applying the least squares method to the exact values of K_t/K_{tN} are expressed as follows. The accurate SCFs are obtained from these equations and K_{tN} given by Equations (4)–(10) with the exponent shown in Equation (11).

Problem (a):

$$
\begin{aligned}
K_t/K_{tN} = {} & (0.9997 - 0.0602\epsilon + 0.4586\epsilon^2) \\
& + (0.4094 - 10.2440\epsilon + 60.4360\epsilon^2)\,\lambda \\
& + (-2.3578 + 69.9210\epsilon - 433.920\epsilon^2)\,\lambda^2 \\
& + (5.2472 - 164.890\epsilon + 1089.0\epsilon^2)\,\lambda^3
\end{aligned}
$$

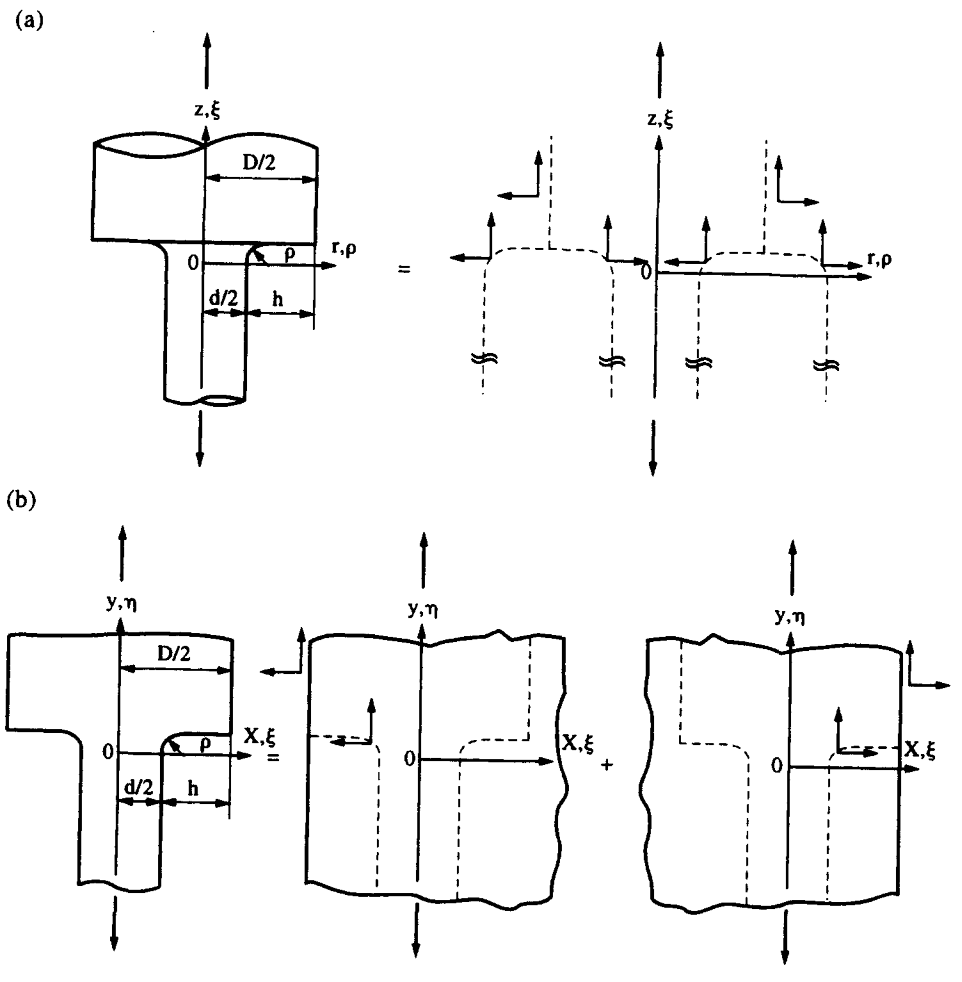

Figure A1 Illustration of analysis of body force method (a) round test specimen, (b) flat test specimen

$$+ (5.8807 + 174.60\epsilon - 1216.1\epsilon^2)\lambda^4$$
$$+ (2.5821 - 69.3580\epsilon + 500.340\epsilon^2)\lambda^5$$
$$(0.03 \leqslant \epsilon \leqslant 0.1,\ 0.02 \leqslant \lambda \leqslant 1.0)$$

$$K_t/K_{tN} = (0.9978 + 0.0063\epsilon - 0.0050\epsilon^2)$$
$$+ (-0.0016 - 0.2320\epsilon + 0.3407\epsilon^2)\lambda$$
$$+ (-0.0098 + 3.3260\epsilon - 3.7404\epsilon^2)\lambda^2$$
$$+ (1.140 - 14.4260\epsilon + 13.2420\epsilon^2)\lambda^3$$
$$+ (-2.8352 + 20.90\epsilon - 17.1370\epsilon^2)\lambda^4$$
$$+ (1.7070 - 9.5675\epsilon + 7.2953\epsilon^2)\lambda^5$$
$$(0.1 < \epsilon \leq 1.0,\ 0.02 \leq \lambda \leq 1.0) \quad \text{(A1)}$$

Problem (b):

$$K_t/K_{tN} = (0.9967 - 0.0163\epsilon + 0.5525\epsilon^2)$$
$$+ (0.9609 - 13.6340\epsilon + 45.0550\epsilon^2)\lambda$$
$$+ (-3.7358 + 42.650\epsilon + 85.3710\epsilon^2)\lambda^2$$
$$+ (7.2784 - 114.50\epsilon - 145.860\epsilon^2)\lambda^3$$
$$+ (-8.7448 + 191.360\epsilon - 374.990\epsilon^2)\lambda^4$$
$$+ (4.2394 - 105.730\epsilon + 389.040\epsilon^2)\lambda^5$$
$$(0.03 \leqslant \epsilon \leq 0.1,\ 0.02 \leqslant \lambda \leqslant 1.0)$$

$$K_t/K_{tN} = (1.0016 - 0.0101\epsilon + 0.0096\epsilon^2)$$
$$+ (0.0486 + 0.2309\epsilon - 0.0889\epsilon^2)\lambda$$
$$+ (1.4471 - 5.7807\epsilon + 2.7451\epsilon^2)\lambda^2$$
$$+ (-5.5762 + 15.3380\epsilon - 5.7006\epsilon^2)\lambda^3$$
$$+ (6.1852 - 12.6180\epsilon + 2.2278\epsilon^2)\lambda^4$$
$$+ (-2.1060 + 2.8386\epsilon + 0.8079\epsilon^2)\lambda^5$$
$$(0.1 < \epsilon \leqslant 1.0,\ 0.02 \leqslant \lambda \leqslant 1.0) \quad \text{(A2)}$$

Problem (c):

$$K_t/K_{tN} = (1.0116 - 0.31986\epsilon + 2.1857\epsilon^2)$$
$$+ (1.3774 - 16.133\epsilon + 52.691\epsilon^2)\lambda$$
$$+ (-9.6183 + 125.25\epsilon - 536.40\epsilon^2)\lambda^2$$
$$+ (23.119 - 332.18\epsilon + 1592.8\epsilon^2)\lambda^3$$
$$+ (-24.537 + 375.26\epsilon - 1901.9\epsilon^2)\lambda^4$$
$$+ (9.6501 - 152.0\epsilon + 791.59\epsilon^2)\lambda^5$$
$$(0.03 \leqslant \epsilon \leqslant 0.1,\ 0.02 \leqslant \lambda \leqslant 1.0)$$

$$K_t/K_{tN} = (1.0017 - 0.00513\epsilon + 0.00310\epsilon^2)$$
$$+ (0.40276 - 1.4920\epsilon + 0.09565\epsilon^2)\lambda$$
$$+ (-3.1788 + 8.8826\epsilon - 5.1412\epsilon^2)\lambda^2$$
$$+ (7.3878 - 17.503\epsilon + 9.6479\epsilon^2)\lambda^3$$
$$+ (-7.5126 + 16.014\epsilon - 8.2227\epsilon^2)\lambda^4$$
$$+ (2.8986 - 5.6953\epsilon + 2.7554\epsilon^2)\lambda^5$$
$$(0.1 < \epsilon \leqslant 1.0,\ 0.02 \leqslant \lambda \leqslant 1.0) \quad \text{(A3)}$$

Problem (d):

$$K_t/K_{tN} = (1.0056 - 0.2429\epsilon + 1.7486\epsilon^2)$$
$$+ (0.4506 - 7.6665\epsilon + 56.3130\epsilon^2)\lambda$$
$$+ (-2.7860 + 79.1550\epsilon - 613.530\epsilon^2)\lambda^2$$
$$+ (6.6344 - 224.150\epsilon + 1840.7\epsilon^2)\lambda^3$$
$$+ (-7.0225 + 225.390\epsilon - 2185.4\epsilon^2)\lambda^4$$
$$+ (2.7199 - 102.560\epsilon + 900.480\epsilon^2)\lambda^5$$
$$(0.03 \leqslant \epsilon \leqslant 0.1,\ 0.02 \leqslant \lambda \leqslant 1.0)$$

$$K_t/K_{tN} = (0.9960 + 0.0036\epsilon - 0.0009\epsilon^2)$$
$$+ (0.3277 - 0.4241\epsilon + 0.2786\epsilon^2)\lambda$$
$$+ (-1.550 + 3.6049\epsilon - 2.4421\epsilon^2)\lambda^2$$
$$+ (3.8632 - 8.0532\epsilon + 4.6901\epsilon^2)\lambda^3$$
$$+ (-4.3296 + 5.3136\epsilon - 1.9942\epsilon^2)\lambda^4$$
$$+ (1.6912 - 0.4402\epsilon - 0.5356\epsilon^2)\lambda^5$$
$$(0.1 < \epsilon \leqslant 1.0,\ 0.02 \leqslant \lambda \leqslant 1.0) \quad \text{(A4)}$$

Problem (e):

$$K_t/K_{tN} = (0.9992 + 0.0261\epsilon - 0.01429\epsilon^2)$$
$$+ (1.2380 - 14.050\epsilon + 66.1690\epsilon^2)\lambda$$
$$+ (-7.2922 + 102.660\epsilon - 505.80\epsilon^2)\lambda^2$$
$$+ (17.0620 - 277.960\epsilon + 1367.9\epsilon^2)\lambda^3$$
$$+ (-18.7010 + 323.960\epsilon - 1572.6\epsilon^2)\lambda^4$$
$$+ (7.6908 - 134.580\epsilon + 644.110\epsilon^2)\lambda^5$$
$$(0.03 \leqslant \epsilon \leqslant 0.1,\ 0.02 \leqslant \lambda \leqslant 1.0)$$

$$K_t/K_{tN} = (1.0 + 0.0057\epsilon - 0.0038\epsilon^2)$$
$$+ (0.4616 + 0.1962\epsilon - 0.4223\epsilon^2)\lambda$$
$$+ (-2.2186 + 1.7423\epsilon - 1.1344\epsilon^2)\lambda^2$$
$$+ (3.9901 - 11.220\epsilon + 10.6210\epsilon^2)\lambda^3$$
$$+ (-3.7398 + 18.5840\epsilon - 17.8460\epsilon^2)\lambda^4$$
$$+ (1.5072 - 9.3041\epsilon + 8.7828\epsilon^2)\lambda^5$$
$$(0.1 < \epsilon \leqslant 1.0,\ 0.02 \leqslant \lambda \leqslant 1.0) \quad \text{(A5)}$$