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Interaction effect of stress intensity factors for any number of collinear interface cracks

NAO-AKI NODA[1] and KAZUHIRO ODA[2]
[1]*Department of Mechanical Engineering, Kyushu Institute of Technology, 1-1 Sensui-cho, Tobata-ku, Kitakyushu 804, Japan*
[2]*Department of Mechanical and Electrical Engineering, Tokuyama College of Technology, 3538 Takajo, Kume, Tokuyama 745, Japan*

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Abstract. In this study, the stress intensity factors for any number of interface cracks are calculated for various spacings, elastic constants and number of cracks and the interaction effect of interface cracks is discussed. The problem is formulated as a system of singular integral equations on the basis of the body force method. In the numerical analysis, the unknown functions of the body force densities which satisfy the boundary conditions are expressed by the products of fundamental density functions and power series. Here, the fundamental density functions are chosen to express the stress field due to a single interface crack exactly. The accuracy of the present analysis is verified by comparing the present results with the results obtained by other researchers and examining the compliance with boundary conditions. The calculation shows that the present method gives rapidly converging numerical results for those problems as well as ordinary crack problems in homogeneous materials. The interaction effect of interface cracks appears in a similar way to ordinary collinear cracks having the same geometrical condition and the maximum stress intensity factor is shown to be linearly related to the reciprocal of number of interface cracks.

Key words: interface crack, stress intensity factor, interaction effect, bonded materials, singular integral equation, body force method.

1. Introduction

In recent years, composite materials and adhesive or bonded joints are being used in wide range of engineering field. The fracture of composites and bonded dissimilar materials is induced mainly from the interfacial region because the angular corner of bonded materials induces singular stress and crack initiation at the interface. Particularly flaws or cracks lying along the interface reduce the strength of the structure significantly. Hence, problem of interface cracks in dissimilar materials is very important from the view point of interface strength and stress analysis of interface cracks have been treated in many papers (Williams, 1959; Erdogan, 1963; Erdogan, 1965; England, 1965; Rice and Sih, 1965; Yuuki and Cho, 1989; Nisitani et al., 1993; Erdogan and Gupta, 1971; Comninou, 1977). In the interface crack problem, it is well known that oscillatory stress singularity and overlapping of crack surfaces appear near the interface crack tip and these are quite different from ordinary cracks in homogeneous material. Therefore, in comparison with the ordinary crack problems, it is difficult to analyze accurately the interface crack problems and there are not enough the data of stress intensity factors for interface cracks.

In the previous papers (Noda and Oda, 1992; Noda and Oda, 1993), the numerical solutions of singular integral equation of the body force method have been already discussed in ordinary crack problems. Then, the numerical method, in which unknown functions are approximated by the products of fundamental density functions and polynomials, has been found to give the

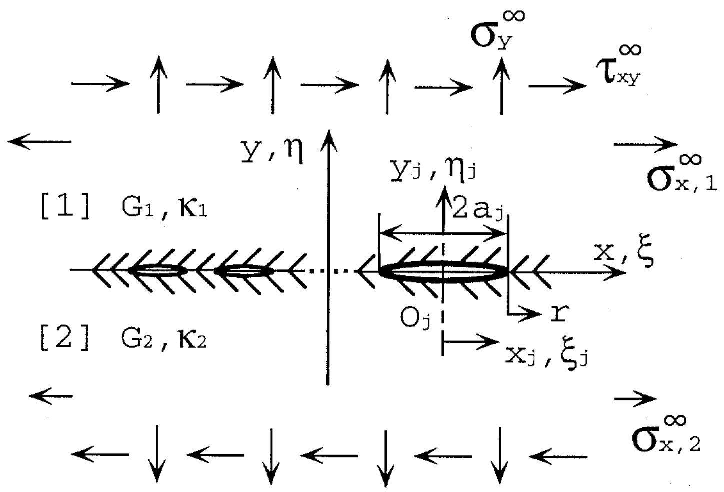

Figure 1. Interaction problem of interface cracks.

results of high accuracy with short CPU time. In addition, the problems of two bonded elastic layers containing a crack perpendicular to and crossing the interface have been treated and the accurate stress intensity factors have been presented (Noda et al., 1989; Noda et al., 1992). In this paper, the method is applied to the analysis of the interaction problem of interface cracks. The calculation is carried out for this problem under various spacings, elastic constants and number of cracks with examining the compliance with boundary conditions. The accurate stress intensity factors to evaluate the interface strength of dissimilar materials are presented in tables and interaction effect of interface cracks is discussed in comparison with the problem of ordinary collinear cracks.

2. Numerical solution of interface cracks

Consider an interaction problem of interface cracks in dissimilar materials as shown in Figure 1. The elastic constants are given as shear modulus and Poisson's ratios for the upper (material 1) and the lower (material 2) half-planes, that is, (G_1, ν_1) and (G_2, ν_2). The remote stresses $\sigma^{\infty}_{x,1}$ and $\sigma^{\infty}_{x,2}$ in x-direction are defined in (1) so as to hold the condition of continuity of strains along the interface, $\varepsilon^{\infty}_{x,1} = \varepsilon^{\infty}_{x,2}$ (Rice and Sih 1965; Isida and Noguchi, 1983)

$$\sigma^{\infty}_{x,2} = \frac{1}{1+\kappa_2}\left[\frac{G_2}{G_1}(1+\kappa_1)\sigma^{\infty}_{x,1} + \left\{3-\kappa_2-\frac{G_2}{G_1}(3-\kappa_1)\right\}\sigma^{\infty}_{y}\right] \tag{1}$$

$$\kappa_m = \begin{cases} (3-\nu_m)/(1+\nu_m) & \text{(Plane stress)} \\ 3-4\nu_m & \text{(Plane strain)} \end{cases} \quad \nu_m : \text{Poisson's ratio } (m = 1, 2), \tag{2}$$

where the subscripts $m = 1$ and 2 refer to material 1 and 2, respectively.

The stress distribution along the interface near a crack tip is expressed by Erdogan (Erdogan, 1963; Erdogan, 1965) and Erdogan and Gupta (1971) as follows,

$$\sigma_y + \tau_{xy} = \frac{K_1 + iK_2}{\sqrt{2\pi r}}\left(\frac{r}{2a_j}\right)^{i\varepsilon} \tag{3}$$

$$\varepsilon = (1/2\pi)\ln\{(G_2\kappa_1 + G_1)/(G_1\kappa_2 + G_2)\}, \tag{4}$$

where K_1 and K_2 are stress intensity factors of an interface crack defined by Yuuki and Cho (1989). The problem can be formulated in terms of singular integral equations by using the stress fields on the interface when two kinds of standard set of force doublets, tension type and shear type, act on a point of interface in dissimilar materials (Nisitani et al., 1993; Saimoto, 1993). The integral equations, which are virtually the boundary condition of jth crack, are expressed as follows,

$$-\pi\beta\frac{\mathrm{d}P_{2j}(x_j)}{\mathrm{d}x_j} + \fint_{-a_j}^{a_j}\frac{P_{1j}(\xi_j)}{(\xi_j - x_j)^2}\,\mathrm{d}\xi_j + \sum_{\substack{k=1\\k\neq j}}^{N}\int_{-a_k}^{a_k} h_y(\xi_k, x_j)P_{1k}(\xi_k)\,\mathrm{d}\xi_k$$

$$= -\sum_{m=1}^{2}\frac{G_m(1+\kappa_m)}{\kappa_m - 1}\frac{\pi}{C}\sigma_y^\infty,$$

$$\pi\beta\frac{\mathrm{d}P_{1j}(x_j)}{\mathrm{d}x_j} + \fint_{-a_j}^{a_j}\frac{P_{2j}(\xi_j)}{(\xi_j - x_j)^2}\,\mathrm{d}\xi_j + \sum_{\substack{k=1\\k\neq j}}^{N}\int_{-a_k}^{a_k} h_{xy}(\xi_k, x_j)P_{2k}(\xi_k)\,\mathrm{d}\xi_k$$

$$= -\sum_{m=1}^{2} G_m\frac{\pi}{C}\tau_{xy}^\infty, \quad (j = 1, \ldots, N) \tag{5}$$

$$C = \frac{2G_1(1+\alpha)}{(1-\beta^2)(\kappa_1 + 1)} = \frac{2G_2(1-\alpha)}{(1-\beta^2)(\kappa_2 + 1)} \tag{6}$$

$$\alpha = \frac{G_2(\kappa_1 + 1) - G_1(\kappa_2 + 1)}{G_2(\kappa_1 + 1) + G_1(\kappa_2 + 1)}, \quad \beta = \frac{G_2(\kappa_1 - 1) - G_1(\kappa_2 - 1)}{G_2(\kappa_1 + 1) + G_1(\kappa_2 + 1)}, \tag{7}$$

where the densities of body force doublets, tension type $P_{1j}(\xi_j)$ and shear type $P_{2j}(\xi_j)$, distributed on the interface are unknown functions and x_j, ξ_j are the coordinate in which the center of jth crack is taken as the origin (Figure 1). The densities $P_{1j}(\xi_j)$ and $P_{2j}(\xi_j)$ are related to the crack opening displacement as shown in later. Equations (5) are virtually the boundary conditions on the interface crack; there are $\sigma_y = 0$ and $\tau_{xy} = 0$. The function $h_y(\xi_k, x_j)$ is stress σ_y induced at the point x_j when the body force doublet with unit density is acting at the imaginary crack site except the jth interface crack. The notation N is the total number of interface cracks, and $\fint$ denotes a finite-part integral proposed by Hadamard (1923) and (Noda and Oda, 1993). For reference, the singular integral equation formulated by means of the continuously distributed dislocation method is shown as follows (Comninou, 1977).

$$-\pi\beta B_{xj}(x_j) + \int_{-a_j}^{a_j}\frac{B_{yj}(\xi_j)}{\xi_j - x_j}\,\mathrm{d}\xi_j + \sum_{\substack{k=1\\k\neq j}}^{N}\int_{-a_k}^{a_k} H_y(\xi_k, x_j)B_{yk}(\xi_k)\,\mathrm{d}\xi_k = -\frac{\pi}{C}\sigma_y^\infty,$$

$$\pi\beta B_{yj}(x_j) + \int_{-a_j}^{a_j}\frac{B_{xj}(\xi_j)}{\xi_j - x_j}\,\mathrm{d}\xi_j + \sum_{\substack{k=1\\k\neq j}}^{N}\int_{-a_k}^{a_k} H_{xy}(\xi_k, x_j)B_{xk}(\xi_k)\,\mathrm{d}\xi_k = -\frac{\pi}{C}\tau_{xy}^\infty, \tag{8}$$

$$\int_{-a_j}^{a_j} B_{xj}(\xi_j)\,\mathrm{d}\xi_j = 0, \quad \int_{-a_j}^{a_j} B_{yj}(\xi_j)\,\mathrm{d}\xi_j = 0. \tag{9}$$

Here $B_{xj}(\xi_j), B_{yj}(\xi_j)$ are densities of edge dislocation. The notation $⨍$ stands for the Cauchy principle value.

In the numerical solution of equations (5), the unknown functions $P_{1j}(\xi_j)$ and $P_{2j}(\xi_j)$ are approximated by the product of the fundamental density functions $w_{1j}(\xi_j), w_{2j}(\xi_j)$ and the weight functions $F_{1j}(\xi_j), F_{2j}(\xi_j)$.

$$P_{1j}(\xi_j) + iP_{2j}(\xi_j) = \{w_{1j}(\xi_j) + iw_{2j}(\xi_j)\}\{F_{1j}(\xi_j) + iF_{2j}(\xi_j)\}. \tag{10}$$

Here, the fundamental density functions are chosen to express the stress field due to a single interface crack exactly. The fundamental density function is derived from the crack opening displacement of the interface crack and expressed as follows (Rice and Sih, 1965; Nisitani et al., 1993):

$$\sum_{m=1}^{2} \left\{ \frac{\kappa_m - 1}{1+\kappa_m} w_{1j}(\xi_j) + iw_{2j}(\xi_j) \right\} = \sum_{m=1}^{2} \frac{1+\kappa_m}{4\cosh \pi\varepsilon} \sqrt{a_j^2 - \xi_j^2} \left(\frac{a_j - \xi_j}{a_j + \xi_j} \right)^{i\varepsilon} . \tag{11}$$

By using the fundamental density function, the crack opening displacements $V_{1j}(\xi_j)$ and $V_{2j}(\xi_j)$ are shown as following equation:

$$V_{1j}(\xi_j) + iV_{2j}(\xi_j) = \sum_{m=1}^{2} \left\{ \frac{\kappa_m - 1}{G_m(1+\kappa_m)} w_{1j}(\xi_j) + i\frac{1}{G_m} w_{2j}(\xi_j) \right\} \{F_{1j}(\xi_j) + iF_{2j}(\xi_j)\}$$

$$V_{1j}(\xi_j) = u_y(\xi_j, +0) - u_y(\xi_j, -0), \quad V_{2j}(\xi_j) = u_x(\xi_j, +0) - u_x(\xi_j, -0), \tag{12}$$

where u_x and u_y denote the displacement in x- and y- directions, respectively.

In this numerical analysis, the weight functions $F_{1j}(\xi_j), F_{2j}(\xi_j)$ are approximated by the following power series.

$$F_{1j}(\xi_j) = \sum_{n=1}^{M} b_n \xi_j^{n-1}, \quad F_{2j}(\xi_j) = \sum_{n=1}^{M} c_n \xi_j^{n-1}. \tag{13}$$

A set of collocation points on jth imaginary crack site is chosen as follows (Noda and Oda, 1992; Kaya and Erdogan, 1987):

$$x_j = a_j \cos\{n\pi/(M+1)\}, \quad (n = 1, \ldots, M), \tag{14}$$

where M is the number of collocation points on jth crack.

By using the numerical method mentioned above, we obtain the $2(M \times N)$ algebraic equations for determining the coefficients b_n and c_n. The stress intensity factor of jth interface crack can be directly calculated from the value of weight function at the crack tip.

$$K_1 + iK_2 = \{F_{1j}(a_j) + iF_{2j}(a_j)\}\sqrt{\pi a_j}(1 + 2i\varepsilon). \tag{15}$$

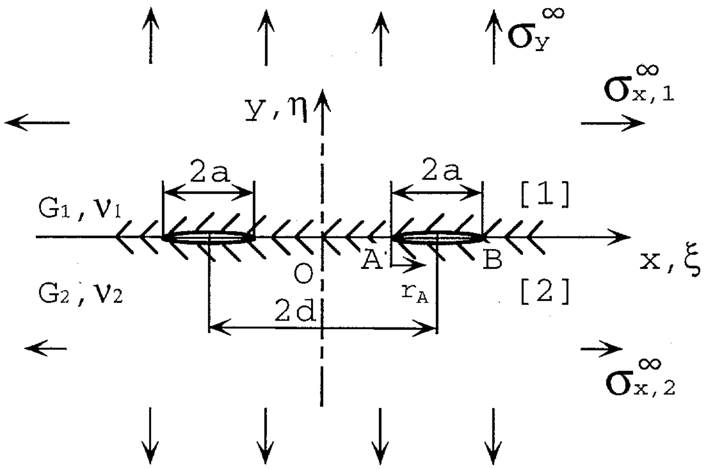

Figure 2. Interaction between two interface cracks with the same length.

Table 1. Convergency of the present result of interface crack $[K_1+iK_2 = (F_1+iF_2)\sigma^\infty\sqrt{\pi a}(1+2i\varepsilon)$. Plane stress, $\Gamma = G_1/G_2 = 0.1, \nu_1 = \nu_2 = 0.3$ in Figure 2]

λ	M	$F_{1,A}$	$F_{2,A}$	$F_{1,B}$	$F_{2,B}$
	4	1.04938	0.00428	1.02881	0.00224
	6	1.04946	0.00431	1.02885	0.00222
	8	1.04946	0.00431	1.02885	0.00220
0.5	10	1.04946	0.00430	1.02886	0.00218
	11	1.04946	0.00430	1.02886	0.00217
	12	1.04946	0.00429	1.02886	0.00216
	13	1.04946	0.00429	1.02886	0.00214
	4	1.22703	0.03631	1.08158	0.01137
	6	1.23281	0.03938	1.08300	0.01087
	8	1.23351	0.03985	1.08319	0.01100
0. 8	10	1.23360	0.03992	1.08321	0.01098
	11	1.23361	0.03992	1.08322	0.01097
	12	1.23361	0.03993	1.08322	0.01096
	13	1.23362	0.03993	1.08322	0.01095

3. Numerical results and discussion

First, the interaction problem of two interface cracks with the same length $2a$ subjected to uniform tension σ_y^∞ in the y-direction is analyzed as shown in Figure 2. The stresses $\sigma_{x,1}^\infty$ and $\sigma_{x,2}^\infty$ at infinity have the relation defined by (1). It is treated under plane stress condition and $\nu_1 = \nu_2 = 0.3$. The stress intensity factors obtained by the present analysis are written by the following dimensionless expressions:

$$K_{1,A} + iK_{2,A} = (F_{1,A} + iF_{2,A})\sigma_y^\infty\sqrt{\pi a}(1 + 2i\varepsilon), \tag{16}$$

where the subscript A refers to the interface crack tip A.

Table 2. Compliance with the boundary conditions near the interface crack tip (r_A: Distance from crack tip A in Figure 2)

$\Gamma = G_1/G_2 = 0.1, \lambda = a/d = 0.8, M = 14$		
r_A/a	σ_y/σ_y^∞	$\tau_{xy}/\sigma_y^\infty$
0.01	−3.4998E-05	7.1568E-05
0.02	−1.4064E-06	3.6386E-06
0.03	2.3276E-06	−7.2903E-06
0.04	2.1751E-06	−8.2153E-06
0.05	1.4159E-06	−6.6027E-06
0.06	7.5789E-07	−4.4652E-06
0.07	3.0787E-07	−2.4732E-06
0.08	8.9595E-08	−8.3326E-07
0.09	−3.1892E-08	3.8552E-07
0.10	−2.5696E-08	1.2284E-06

To verify the accuracy of the present numerical method, we examine the convergency of the present results and the compliance with the boundary conditions. Table 1 shows the convergency of the results with increasing the collocation number M when $\lambda = a/d = 0.5$ and 0.8, where $2d$ is the distance between midpoint of both cracks. The present results have the convergency in almost 5 digits when $M = 12$. Table 2 shows the values of σ_y/σ_y^∞ and $\tau_{xy}/\sigma_y^\infty$ near the crack tip when $\lambda = 0.8$. These values which should be zero along the crack surface are less than 10^{-5} even when $M = 14$. Therefore it is found that the present numerical method is useful to analyze the interface crack problem as well as ordinary crack problem in homogeneous material.

In Table 3, dimensionless stress intensity factors obtained by the present method are shown when the values of $\Gamma = G_1/G_2$ and $\lambda = a/d$ are changed systematically. The parenthesized value in Table 3 is the results of Saimoto (1993) and the asterisked value is the exact solution for the two collinear cracks with the same length in homogeneous material given by Erdogan (1962). As shown in Table 3, present results are in good agreement with the results of Saimoto and especially with Erdogan's exact solution when $\Gamma = 1.0$.

Next, Table 4 shows the result of three interface cracks with the same length as illustrated in Figure 3. The parenthesized value in Table 4 is the exact solution for the three collinear cracks with the same length in homogeneous material given by Sih (1964). The present results are in close agreement with the exact solution in wide range of λ when $\Gamma = 1.0$. Table 5 shows the dimensionless stress intensity factors for two interface cracks with the same length when the values of Poisson's ratio ν_1 and ν_2 are changed. From Table 5, it is indicated that the effect of Poisson's ratio on F_1-value of interface crack is small.

From Tables 3, 4 and 5, F_1-values are found to be almost constant in a wide range of the ratio $\Gamma = G_1/G_2 = 0.001 \sim 1.0$ and Poisson's ratio $\nu_1, \nu_2 = 0 \sim 0.4$. Then, it seems that F_1-value of several interface cracks defined by equation (3) can be approximately estimated from the results of collinear cracks in homogeneous material.

Finally, the stress intensity factors for any number of interface cracks with the same length are analyzed as illustrated in Figure 4. Tables 6 and 7 show the maximum values of stress intensity factors, $F_{1,\max}$ and $F_{2,\max}$, when the values of the number of interface cracks

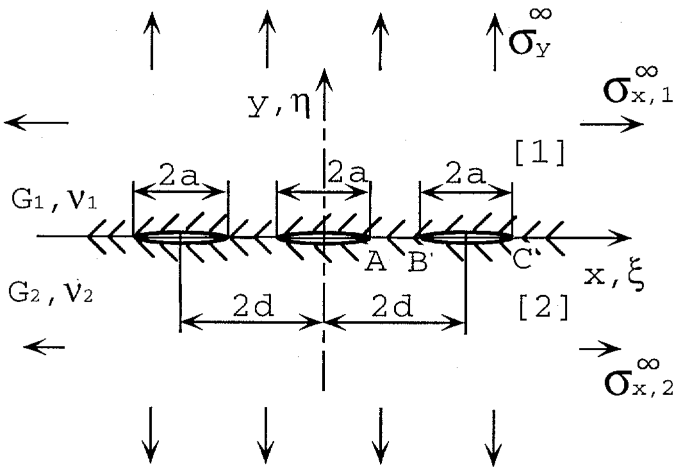

Figure 3. Interaction between three interface cracks with the same length.

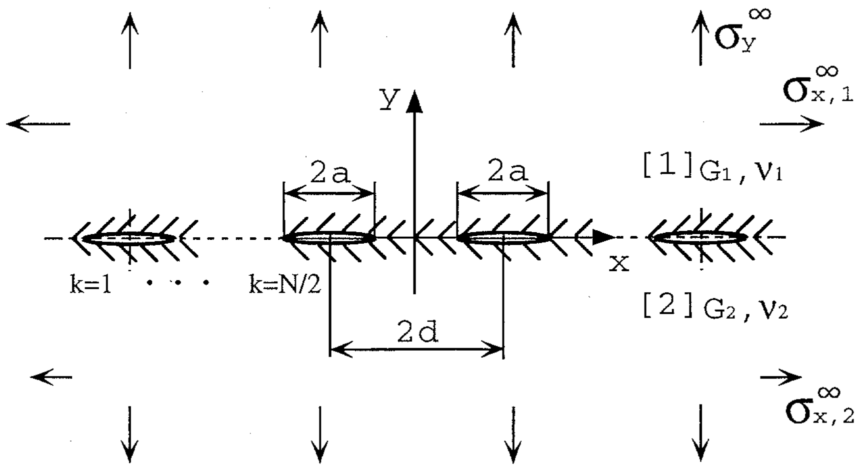

Figure 4. Row of interface cracks with the same length (When the number of cracks is an even number).

$N, \lambda = a/d$ and $\Gamma = G_1/G_2$ are changed systematically. As shown in these tables, it is found that the values of $F_{1,\max}$ increase with increasing of the number of cracks N; on the other hand, the values of $F_{2\max}$ converge when $N = 2 \sim 5$. In this analysis, $F_{1,\max}$ appears at the middle crack and $F_{2\max}$ occurs at the outermost interface crack. Figure 5 shows $F_{1,\max} - 1/N$ relations when $\Gamma = G_1/G_2 = 0.1$ and $\lambda = a/d = 1/3, 1/2, 2/3$. From this figure, the values of $F_{1,\max}$ are found to be nearly linear with $1/N$ for fixed value of $\lambda = a/d$. In Tables 6, 7 and Figure 5, the limiting values for $N \to \infty$ are extrapolated from the values for $N = 9$ and $N = 10$ by using the linearity between $F_{1\max}$ and $1/N$. These results are similar to the problem of collinear cracks (Isida and Igawa, 1993).

Table 3. Dimensionless SIFs F_1 and F_2 for two interface cracks with the same length 2a $[K_1 + iK_2 = (F_1 + iF_2)\sigma^{\infty}\sqrt{\pi a}(1 + 2i\varepsilon)$, $\Gamma = G_1/G_2, \lambda = a/d$, Plane stress, $\nu_1 = \nu_2 = 0.3$ in Figure 2]

Γ	λ	0.1		0.2		0.5		2/3		0.8	
1.0	$F_{1,A}$	1.00132	[1.00132]*	1.00566	[1.00566]*	1.04796	[1.04796]*	1.1124		1.2289	[1.22894]*
	$F_{1,B}$	1.00119	[1.00120]*	1.00462	[1.00462]*	1.02795	[1.02795]*	1.0516		1.0810	[1.08107]*
0.1	$F_{1,A}$	1.0013	(1.0014)	1.0058	(1.0058)	1.0494	(1.0489)	1.1155	(1.1145)	1.2336	
	$F_{2,A}$	0.0000	(0.0000)	0.0001	(0.0001)	0.0042	(0.0035)	0.0146	(0.0118)	0.0399	
	$F_{1,B}$	1.0012	(1.0012)	1.0047	(1.0047)	1.0288	(1.0285)	1.0532	(1.0527)	1.0832	
	$F_{2,B}$	0.0000	(0.0000)	0.0001	(0.0001)	0.0021	(0.0018)	0.0055	(0.0045)	0.0109	
0.01	$F_{1,A}$	1.0013	(1.0014)	1.0059	(1.0059)	1.0501	(1.0493)	1.1169	(1.1154)	1.2357	
	$F_{2,A}$	0.0000	(0.0000)	0.0002	(0.0002)	0.0052	(0.0042)	0.0180	(0.0145)	0.0490	
	$F_{1,B}$	1.0012	(1.0012)	1.0048	(1.0048)	1.0292	(1.0288)	1.0539	(1.0531)	1.0842	
	$F_{2,B}$	0.0000	(0.0000)	0.0001	(0.0001)	0.0026	(0.0022)	0.0068	(0.0055)	0.0134	
0.001	$F_{1,A}$	1.0013	(1.0014)	1.0059	(1.0059)	1.0502	(1.0934)	1.1171	(1.1154)	1.2360	
	$F_{2,A}$	0.0000	(0.0000)	0.0002	(0.0002)	0.0054	(0.0043)	0.0184	(0.0146)	0.0501	
	$F_{1,B}$	1.0012	(1.0012)	1.0048	(1.0048)	1.0293	(1.0288)	1.0540	(1.0532)	1.0843	
	$F_{2,B}$	0.0000	(0.0000)	0.0001	(0.0001)	0.0027	(0.0022)	0.0070	(0.0056)	0.0137	

( ): Saimoto (1993); [ ]*: Erdogan (1962).

Table 4. Dimensionless SIFs F_1 and F_2 for three interface cracks with the same length 2a $[K_1 + iK_2 = (F_1 + iF_2)\sigma^\infty \sqrt{\pi a}(1 + 2i\varepsilon), \Gamma = G_1/G_2, \lambda = a/d$, Plane stress, $\nu_1 = \nu_2 = 0.3$ in Figure 3]

Γ	λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1.0	$F_{1,A}$	1.00251	1.01029	1.02407	1.04529	1.07663	1.12316	1.19557	1.3213	1.606
		(1.00252)	(1.01030)	(1.02407)	(1.04529)	(1.07663)	(1.12316)	(1.19558)	(1.32136)	(1.60685)
	$F_{1,B}$	1.00164	1.00701	1.01710	1.03353	1.05913	1.09915	1.16456	1.2834	1.564
		(1.00164)	(1.00702)	(1.01710)	(1.03353)	(1.05913)	(1.09915)	(1.16456)	(1.28348)	(1.56454)
	$F_{1,C}$	1.00150	1.00585	1.01296	1.02297	1.03631	1.05383	1.07724	1.1103	1.164
		(1.00150)	(1.00585)	(1.01296)	(1.02297)	(1.03631)	(1.05383)	(1.07724)	(1.11032)	(1.16439)
0.1	$F_{1,A}$	1.0026	1.0106	1.0248	1.0468	1.0790	1.1269	1.2010	1.3290	1.616
	$F_{2,A}$	0.0000	0.0000	0.0002	0.0008	0.0021	0.0052	0.0121	0.0295	0.086
	$F_{1,B}$	1.0016	1.0072	1.0176	1.0346	1.0610	1.1022	1.1692	1.2900	1.570
	$F_{2,B}$	0.0000	0.0001	0.0005	0.0019	0.0048	0.0104	0.0220	0.0484	0.128
	$F_{1,C}$	1.0015	1.0060	1.0134	1.0237	1.0375	1.0556	1.0796	1.1136	1.168
	$F_{2,C}$	0.0000	0.0001	0.0002	0.0010	0.0024	0.0046	0.0081	0.0140	0.025
0.01	$F_{1,A}$	1.0026	1.0108	1.0252	1.0475	1.0802	1.1286	1.2036	1.3326	1.620
	$F_{2,A}$	0.0000	0.0000	0.0003	0.0010	0.0026	0.0064	0.0149	0.0363	0.106
	$F_{1,B}$	1.0017	1.0073	1.0179	1.0351	1.0619	1.1037	1.1714	1.2930	1.573
	$F_{2,B}$	0.0000	0.0002	0.0006	0.0024	0.0059	0.0129	0.0272	0.0596	0.158
	$F_{1,C}$	1.0015	1.0061	1.0136	1.0241	1.0381	1.0564	1.0808	1.1151	1.170
	$F_{2,C}$	0.0000	0.0001	0.0003	0.0012	0.0029	0.0057	0.0101	0.0173	0.031
0.001	$F_{1,A}$	1.0026	1.0108	1.0253	1.0476	1.0804	1.1289	1.2039	1.3330	1.621
	$F_{2,A}$	0.0000	0.0000	0.0003	0.0010	0.0027	0.0065	0.0153	0.0371	0.108
	$F_{1,B}$	1.0017	1.0073	1.0179	1.0352	1.0620	1.1038	1.1716	1.2934	1.573
	$F_{2,B}$	0.0000	0.0002	0.0006	0.0024	0.0060	0.0132	0.0277	0.0609	0.161
	$F_{1,C}$	1.0015	1.0061	1.0136	1.0242	1.0382	1.0565	1.0810	1.1154	1.170
	$F_{2,C}$	0.0000	0.0001	0.0003	0.0013	0.0030	0.0058	0.0102	0.0177	0.032

( ): Sih (1964).

Table 5. F_1 and F_2 for two interface cracks when Poisson's ratio is changed (Plane stress, $\Gamma = G_1/G_2 = 0.001$, $\lambda = a/d = 0.8$ in Figure 2)

ν_1	ν_2	$F_{1,A}$	$F_{2,A}$	$F_{1,B}$	$F_{2,B}$
	0.0	1.2444	0.0792	1.0883	0.0217
	0.1	1.2444	0.0792	1.0883	0.0217
0.0	0.2	1.2444	0.0792	1.0883	0.0217
	0.3	1.2444	0.0792	1.0883	0.0217
	0.4	1.2444	0.0792	1.0883	0.0217
	0.0	1.2411	0.0686	1.0868	0.0188
	0.1	1.2411	0.0686	1.0868	0.0188
0.1	0.2	1.2411	0.0686	1.0868	0.0188
	0.3	1.2411	0.0686	1.0868	0.0188
	0.4	1.2411	0.0686	1.0868	0.0188
	0.0	1.2384	0.0590	1.0854	0.0161
	0.1	1.2384	0.0590	1.0854	0.0161
0.2	0.2	1.2384	0.0590	1.0854	0.0161
	0.3	1.2384	0.0590	1.0854	0.0161
	0.4	1.2384	0.0590	1.0854	0.0161
	0.0	1.2360	0.0502	1.0843	0.0137
	0.1	1.2360	0.0502	1.0843	0.0137
0.3	0.2	1.2360	0.0502	1.0843	0.0137
	0.3	1.2360	0.0502	1.0843	0.0137
	0.4	1.2360	0.0502	1.0843	0.0137
	0.0	1.2340	0.0420	1.0834	0.0115
	0.1	1.2340	0.0420	1.0834	0.0115
0.4	0.2	1.2340	0.0420	1.0834	0.0115
	0.3	1.2340	0.0420	1.0834	0.0115
	0.4	1.2340	0.0420	1.0834	0.0115

Table 6. Maximum SIFs for any number of interface cracks with the same length 2a (Plane stress, $\Gamma = G_1/G_2 = 0.1$, $\nu_1 = \nu_2 = 0.3$ in Figure 4)

N \ a/d	$F_{1,\max}$			$F_{2,\max}$		
	1/3	1/2	2/3	1/3	1/2	2/3
2	1.018	1.049	1.116	0.001	0.004	0.015
3	1.031	1.079	1.172	0.001	0.005	0.017
4	1.036	1.091	1.200	0.001	0.005	0.018
5	1.039	1.100	1.220	0.001	0.005	0.018
6	1.041	1.105	1.232	0.001	0.005	0.019
7	1.043	1.109	1.241	0.001	0.005	0.019
8	1.044	1.112	1.248	0.001	0.005	0.019
9	1.045	1.114	1.253	0.001	0.005	0.019
10	1.045	1.116	1.258	0.001	0.005	0.019
∞	1.051	1.132	1.296	0.001	0.005	0.019

Table 7. Maximum SIFs for any number of interface cracks with the same length 2a (Plane stress, $\Gamma = G_1/G_2 = 0.01, \nu_1 = \nu_2 = 0.3$ in Figure 4)

N \ a/d	$F_{1,\max}$			$F_{2,\max}$		
	1/3	1/2	2/3	1/3	1/2	2/3
2	1.018	1.050	1.117	0.001	0.005	0.018
3	1.032	1.080	1.175	0.001	0.006	0.021
4	1.036	1.092	1.203	0.001	0.006	0.022
5	1.040	1.101	1.223	0.001	0.006	0.023
6	1.042	1.107	1.235	0.001	0.006	0.023
7	1.043	1.111	1.245	0.001	0.006	0.023
8	1.045	1.114	1.252	0.001	0.006	0.023
9	1.046	1.116	1.257	0.001	0.006	0.023
10	1.046	1.118	1.262	0.001	0.006	0.023
∞	1.052	1.134	1.301	0.001	0.006	0.023

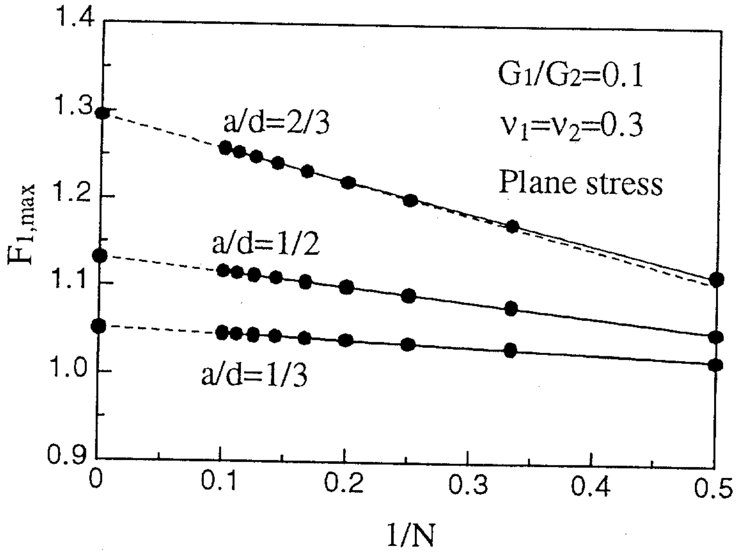

Figure 5. An example of $F_{1,\max} - 1/N$ relations (Plane stress, $\Gamma = G_1/G_2 = 0.1, \nu_1 = \nu_2 = 0.3$).

4. Conclusions

In this paper, the stress intensity factors for any number of interface cracks were calculated very accurately by using the singular integral equations of the body force method and the interaction effects of interface cracks were considered. The conclusions can be made as follows.

(1) Singular integral equations of the body force method were shown in comparison with the ones of the continuously distributed dislocation method. In the numerical solution, the unknown functions were approximated by the fundamental density functions and power series. Here, the fundamental density functions were chosen to express the stress field due to a single interface crack exactly. It was found that the method gave rapidly converging numerical results and highly satisfied boundary conditions.

(2) The dimensionless stress intensity factors defined by equation (1) for any number of interface cracks were shown in tables for various spacing $\lambda = a/d$ and elastic parameters

$\Gamma = G_1/G_2$. The results of two and three interface cracks with the same length were in close agreement with the exact solution in homogeneuous material when $G_1 = G_2$.

(3) F_1-values were found to be almost constant in a wide range of the ratio of elastic constants $\Gamma = G_1/G_2 = 0.001 \sim 1.0$ and Poisson's ratio $\nu_1, \nu_2 = 0 \sim 0.4$. Then, it is possible that F_1-value of several interface cracks can be approximately estimated from the results of collinear cracks in homogeneous material.

(4) The maximum stress intensity factors $F_{1,\max}$ for any number of interface cracks were found to be nearly linear with $1/N$ (N: number of interface cracks) for fixed value of $\lambda = a/d$. From these results, it was found that the interaction effect of interface cracks appeared in a similar way to ordinary collinear cracks having the same geometrical condition.

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