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Effect of Shape and Arrangement of Inclusions on the Elastic Modulus of Composite Materials

N.-A. Noda[1], H. Nisitani[2], Y. Takase[1] and T. Wada[1]

[1] Department of Mechanical Engineering, Kyushu Institute of Technology, Kitakyushu 804, Japan

[2] Department of Mechanical Engineering, Kyushu Sangyo University, Fukuoka 813, Japan

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ABSTRACT

In this paper, the effect of shape and arrangement of inclusions on the elastic modulus of composite materials is considered when equally shaped inclusions are arranged in matrix. Effective elastic modulus of the composite materials are numerically analyzed by the application of FEM. First, periodically arranged 2D inclusions are considered. From the comparison between rectangular and elliptical inclusions, it is found that the projected width of inclusions in the tensile direction and the volume fraction of inclusion are two major parameters controlling the effective elastic modulus of composites. Next, an effect of arrangements of rectangular inclusions on the elastic modulus of composite material is considered. Finally, from the comparison between the results of cylindrical and rectangular inclusions, it is found that 3D arrangement of inclusions can be evaluated by analyzing 2D arrangement of rectangular inclusions that has the same values of the projected area and volume fraction of inclusions.

1. INTRODUCTION

Mechanical properties of composite materials are being investigated in wide areas [1-5]. The effective elastic modulus is usually evaluated by the concept of law of mixture. However, the detailed effect of the shape and arrangement of inclusions upon the effective elastic modulus has not been clarified yet. To design composite materials, it is practically important to predict the elastic modulus when the shape and arrangement of inclusions are given in the matrix. From this viewpoint, the effect of shape and arrangement of inclusions on the elastic modulus of composite material is considered in this study by the application of FEM when equally shaped inclusions are arranged in matrix. In addition, periodically arranged inclusions in a 3D body are also considered. Then, the dominant factor of the shape of the inclusion on the elastic modulus of composite materials will be discussed.

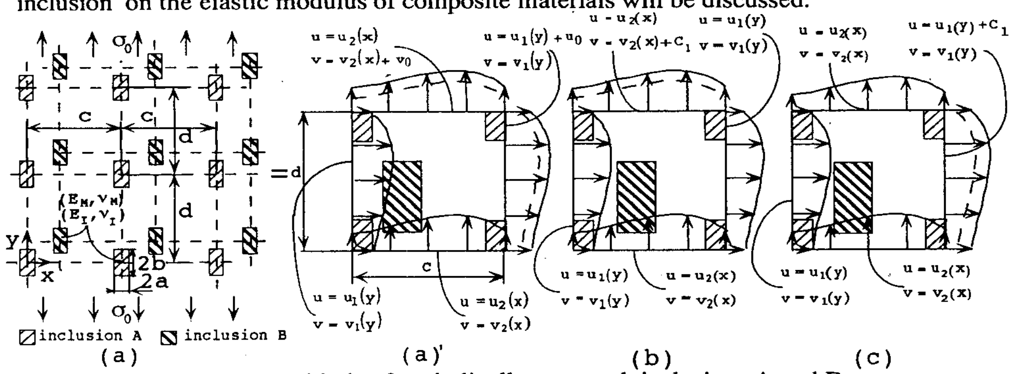


Fig.1 Two kinds of periodically arranged inclusions A and B

2. METHOD OF ANALYSIS

In this section, using a model as shown in Fig.1 the method of analysis will be explained. Here, the elastic modulus is considered when rectangular inclusions A are fixed and the location of inclusions B varies. These two kinds of periodically arranged inclusions A and B shown in Figure 1 (a) can be analyzed by using FEM as shown in the following procedure. Figure 1 (a)' is regarded as the unit cell region of Figure 1 (a). Then, the boundary conditions can be expressed as follows. Here, u,v are displacements in the x, y-direction, respectivery.

$$\left.\begin{aligned}
&(\mathrm{I})\ \text{If } u = u_1(y),\quad v = v_1(y),\ \text{when } x=0,\ 0\leqq y\leqq d \\
&\quad\ \ \text{then } u = u_1(y)+u_0,\quad v = v_1(y),\ \text{when } x=c,\ 0\leqq y\leqq d \\
&(\mathrm{II})\ \text{If } u = u_2(x),\quad v = v_2(x),\ \text{when } y=0,\ 0\leqq x\leqq c \\
&\quad\ \ \text{then } u = u_2(x),\quad v = v_2(x)+v_0,\ \text{when } y=d,\ 0\leqq x\leqq c
\end{aligned}\right\} \quad \cdots\cdots\cdots (1)$$

And

$$\int_0^c \sigma_y\big|_{y=0,\,d}\,dx = \sigma_0\times c,\quad \int_0^d \sigma_x\big|_{x=0,\,c}\,dy = 0 \quad \cdots\cdots\cdots (2)$$

In eqn (1), u_0, v_0 are unknown constants. Therefore, the following method will be applied. First, under the boundary condition as shown in eqn(3) the problem of Figure 1(b) is analyzed. Here, this C_1 is an arbitrary constant.

$$\left.\begin{aligned}
&(\mathrm{I})\ u = u_1(y),\quad v = v_1(y) \\
&\quad\ \ \text{when } x=0,\ 0\leqq y\leqq d,\ \text{and } x=c,\ 0\leqq y\leqq d \\
&(\mathrm{II})\ \text{If } u = u_2(x),\quad v = v_2(x),\ \text{when } y=0,\ 0\leqq x\leqq c \\
&\quad\ \ \text{then } u = u_2(x),\quad v = v_2(x)+C_1,\ \text{when } y=d,\ 0\leqq x\leqq c
\end{aligned}\right\} \quad \cdots\cdots\cdots (3)$$

Under the condition (3), the resultant force F_1 in the x-direction on the boundaries x=0, c with $0\leqq y\leqq d$ and the resultant force F_2 in y-direction on the boundaries y=0, d with $0\leqq x\leqq c$ are calculated as shown in eqn(4).

$$\int_0^d \sigma_x\big|_{x=0,\,c}\,dy = F_1,\quad \int_0^c \sigma_y\big|_{y=0,\,d}\,dx = F_2 \quad \cdots\cdots\cdots (4)$$

Next, under the boundary condition as shown in eqn(5) the problem of Figure 1(c) is analyzed.

$$\left.\begin{aligned}
&(\mathrm{I})\ \text{If } u = u_1(y),\quad v = v_1(y),\ \text{when } x=0,\ 0\leqq y\leqq d \\
&\quad\ \ \text{then } u = u_1(y)+C_1,\quad v = v_1(y)\ \text{when } x=c,\ 0\leqq y\leqq d \\
&(\mathrm{II})\ u = u_2(x),\quad v = v_2(x) \\
&\quad\ \ \text{when } y=0,\ 0\leqq x\leqq c\ \text{and } y=d,\ 0\leqq x\leqq c
\end{aligned}\right\} \quad \cdots\cdots\cdots (5)$$

Under the condition (5), the resultant force F_3 in the x-direction on the boundaries x=0, c with $0\leqq y\leqq d$ and the resultant force F_4 in the y-direction on the boundaries y=0, d with $0\leqq x\leqq c$ are calculated as shown in eqn(6).

$$\int_0^d \sigma_x\big|_{x=0,\,c}\,dy = F_3,\quad \int_0^c \sigma_y\big|_{y=0,\,d}\,dx = F_4 \quad \cdots\cdots\cdots (6)$$

The solution for Figure 1 (a) can be expressed by superposing the solution for Figure 1 (b) and the solution for Figure 1 (c) as shown in eqn (7). Here the solutions of Figure 1 (a)', (b), (c) denote (σ_a, u_a), (σ_b, u_b), (σ_c, u_c), respectively.

$$\left.\begin{aligned} &\sigma_a = A\sigma_b + B\sigma_c, \quad u_a = Au_b + Bu_c \\ &A = \frac{\sigma_0 c}{F_2 - F_4(F_1/F_3)}, \quad B = -\frac{(F_1/F_3)\sigma_0 c}{F_2 - F_4(F_1/F_3)} \end{aligned}\right\} \quad \cdots\cdots\cdots (7)$$

Here, this A, B are constant defined in eqn (8).

$$\left.\begin{aligned} &A \times F_1 + B \times F_3 = 0 \\ &A \times F_2 + B \times F_4 = \sigma_0 c \end{aligned}\right\} \quad \cdots\cdots\cdots (8)$$

The constant displacement u_0, v_0 in Figure 1(a) can be expressed as follows.

$$u_0 = Bc_1, \quad v_0 = Ac_1 \quad \cdots\cdots\cdots (9)$$

The effective elastic constants of the composite shown in Figure 1(a) are given by equation (10).

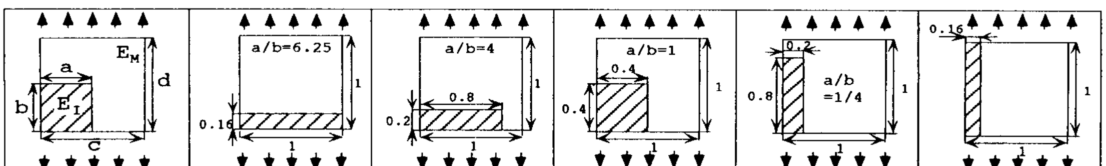
$$\left.\begin{aligned} &E = \frac{\sigma_0}{(v_0/d)} = \frac{\{F_2 - F_4(F_1/F_3)\}/c}{c_1/d} \\ &\nu = \frac{u_0/c}{v_0/d} = \frac{F_3 d}{F_1 c} \end{aligned}\right\} \quad \cdots\cdots\cdots (10)$$

3. RESULTS AND DISCUSSION

First, in Fig.1 only periodically arranged 2D inclusions A whose sizes $a \times b$ are assumed in matrix without inclusions B. Then, the effect of the shape of inclusions A on the effective elastic modulus E is considered. Plane stress condition with Poisson's ratio $\nu = 0.3$ is assumed in the following calculations. Here, E_I and E_M are elastic modulus of inclusion and matrix, respectively. In the present analysis, quadrilateral 4 node elements are used. The analysis is made by using MARC K52 and pre/postprocessor MENTAT2.

In Table 1, the values obtained by an approximate formula (11), which is known as an extended law of mixture, are shown and compared with the FEM results under constant volume fraction of inclusion V_I=0.16. Here the unit cell has the dimensions $c \times d = 1 \times 1$; then, the volume fraction of inclusion is V_I=ab for the rectangular inclusion.

Table 1 Results of rectangular rigid inclusions when ab/(cd)=0.16



E/E_M	1.19	1.24	1.37	2.30	16001
E^*/E_M	1.19	1.20	1.27	1.80	16001

$$E^*/E_M = \frac{a}{c}\frac{d}{\left(\frac{b}{E_I}+\frac{d-b}{E_M}\right)} + \frac{(c-a)/c}{E_M} \qquad \cdots\cdots\cdots(11)$$

Table 1 indicates that the effective elastic modulus varies depending on the shape of inclusion a/b even though the volume fraction of inclusion is constant. In Table 1, as b/d →1, eqn(1) has large error.

In Fig.2, the elastic modulus of the rectangular inclusions is compared with the one of the elliptical inclusions [5]. When the unit cell has the dimensions c×d=1×1, the volume fraction for elliptical inclusion is $V_I=\pi a'b'/4$, where a', b' are radii of ellipse. Fig.2 indicates that the effective elastic modulus E varies depending on the width of inclusion a/c even though the volume fraction of inclusion is constant. Through the comparison between the present results and the ones of elliptical inclusions, the elastic moduli of different shaped inclusions are nearly equal under the following conditions:

(1) the projected widths of inclusions are equal, that is, a=a'

(2) the volume fractions are equal, that is, ab= π a'b'/4.

Figure 3 (a), (b) illustrates the equivalent conditions of inclusions. If the conditions are satisfied, it can be concluded that the effects of shape of inclusion on the effective elastic modulus are almost equivalent even though the shape of inclusions differs from rectangle or ellipse.

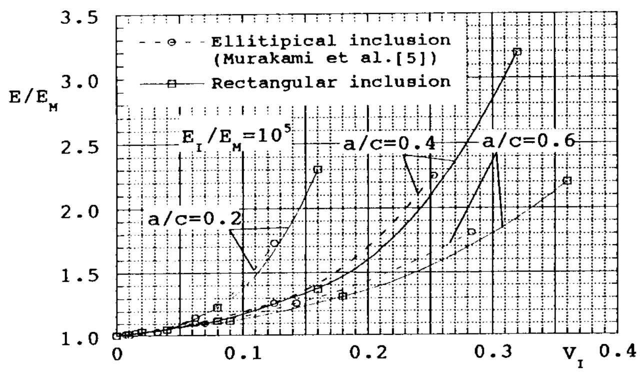


Fig.2 E/E_M vs V_I relation when a/c=0.2, 0.4, 0.6

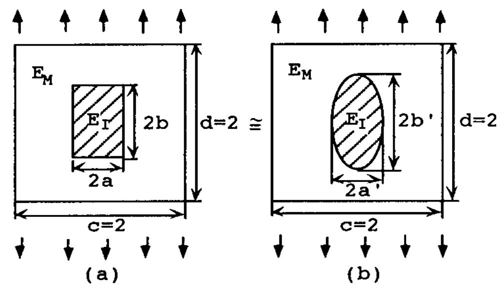


Fig.3 Elastic modulus is almost equal on the conditions that (1) a=a' and (2) ab= π a'b'/4

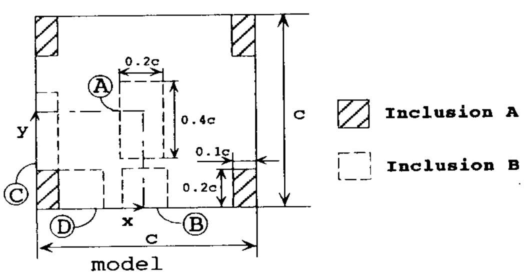


Fig.4 Effect of inclusion B when inclusion A is fixed

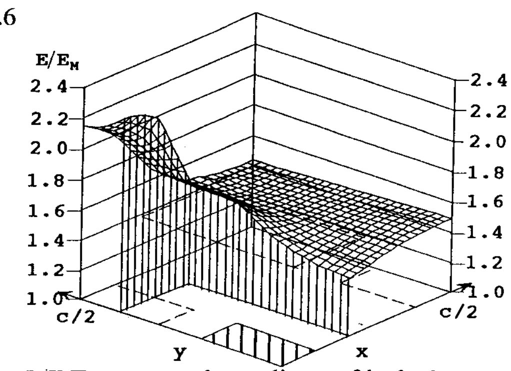


Fig.5 E/E_M vs central coordinate of inclusion B in model Fig.1(a) (b/a=2, V_I =0.16, $E_I/E_M=10^5$)

Next, the effect of arrangement of rectangular inclusions on the elastic modulus E of composite material is considered in the model of Fig.1 when rectangular inclusions A are fixed and the location of rectangular inclusions B varies. Here, the volume fraction of inclusions (A and B) is V_I=8ab in the unit cell as shown in Fig.4. In Fig.4 the shape and elastic ratio of inclusion are assumed as a/b=0.5 and $E_I/E_M=10^5$. Figure 5 expresses that the efective elastic constants with varing the position of inclusions B in the x, y plane when inclusions A are fixed. The maximum value of E/E_M is 2.286 at point C in Fig.4 and the minimum value of E/E_M is 1.384 at point D in Fig.4.

Finally, periodically arranged inclusions in a 3D body as shown in Fig.6 are considered. The unit cell is apprpximated by a cylindrical shape as shown in Fig.6 (c). Here, the volume fraction of inclusion is $V_I=a^{*2}b^*$ for cylindrical inclusion in the unit cell in Fig.8, where a* and b* are radius and hight of the cylindrical inclusion [see unit cell in Fig 8.(b)]. In Figure 7(a), (b), the elastic modulus of the cylindrical inclusion is compared with the rectangular inclisions in 2D array. It is found that the elastic moduli of 2D and 3D arrangement of inclusions, in other words, rectangular and cylindrical inclusions are nearly equal under the following conditions:

(1) the projected areas of inclusions are equal, that is, $a/c=(a^*/c^*)^2$

(2) the volume fractions are equal, that is, $ab/(cd)=a^{*2}b^*/(c^{*2}d^*)$.

It is found that 3D arrangement may be evaluated by analyzing 2D arrangement that has the same values of the projected area and volume fraction of inclusions.

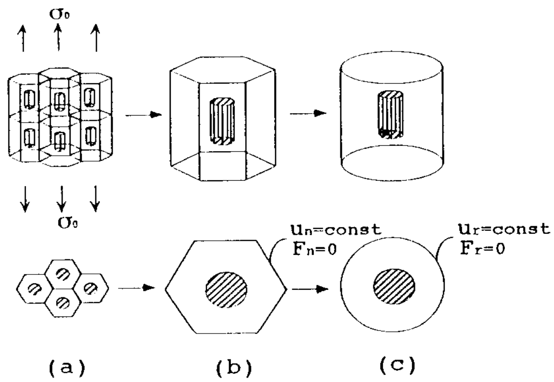


Fig.6 Model for 3D array of inclusions

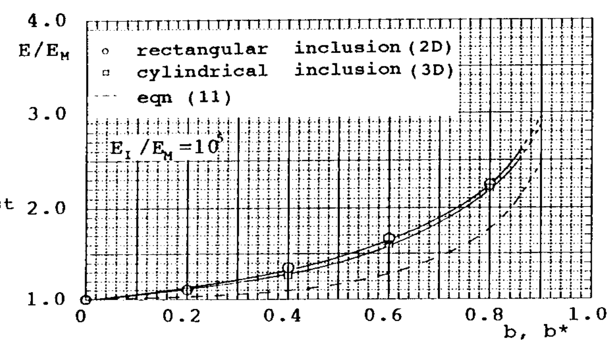


Fig.7(a) E/E_M vs b, b* relation [a/c=0.2, $(a^*/c^*)^2=(0.45)^2=0.2025$ in Fig.8]

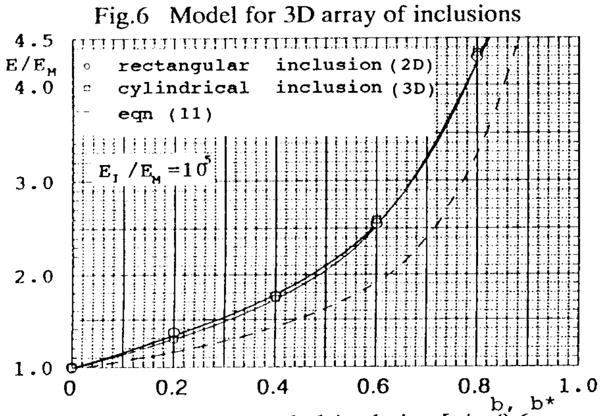


Fig.7(b) E/E_M vs b, b* relation [a/c=0.6, $(a^*/c^*)^2=(0.75)^2=0.5625$ in Fig.8]

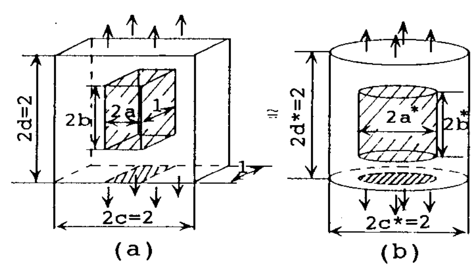


Fig.8 Elastic modulus is almost equal on the condition that (1) $a/c=(a^*/c^*)^2$ and (2) $ab/(cd)=a^{*2}b^*/(c^{*2}d^*)$

4. CONCLUSION

In this paper, the effect of shape and arrangement of inclusions on the elastic modulus of composite material is considered when equally shaped inclusions are arranged in matrix. Elastic modulus of the composite material are numerically analyzed by the application of FEM. The conclusions can be made as follows.

(1) In periodically arranged 2D rectangular inclusions, the effective elastic modulus varies depending on the shape of inclusion a/b even though the volume fraction of inclusion is constant. An extended law of mixture eqn(1) sometimes has large error. Results of elliptical and rectangular inclusions are compared. Then, it is found that the projected width and volume fraction of inclusions are two major parameters controlling the effective elastic modulus of composite materials. In real composites, the shape of inclusion may be approximated by ellipse; therefore, the effective elastic constant may be calculated accurately and conveniently by analyzing rectangular inclusions having the same projected width and volume fraction of real inclusions. It should be noted FEM mesh division is especially easy for rectangular inclusions.

(2) Effect of arrangements of rectangular inclusions on the elastic modulus of composite material is considered when periodic arrangement of rectangular inclusions A are fixed and the location of periodic arrengement of rectangular inclusions B varies. It is found that if the projected width and volume fraction of inclusions are equal each other for two different composites the elastic moduli of these composites are nearly equal.

(3)Periodically arranged inclusions in a 3D body as shown in Fig.6 are also considered. The elastic modulus of the cylindrical inclusions in 3D array is compared with the one of rectangular inclisions in 2D array. It is found that the elastic moduli of 2D and 3D arrangement of inclusions, in other words, rectangular and cylindrical inclusions as shown in Fig.8, are nearly equal under the following conditions:

(1) the projected areas of inclusions are equal, that is, $a/c=(a^*/c^*)^2$

(2) the volume fractions are equal, that is, $ab/(cd)=a^{*2}b^*/(c^{*2}d^*)$.

It is found that 3D arrangement may be evaluated by the replacement to 2D arrangement that has the same values of the projected area and volume fraction of inclusions.

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