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Two types of optimization of a cylindrical shell subjected to lateral pressure

Bin Liang [a,b,*], N. Noda [b,1], Shufen Zhang [a,2]

[a] College of Architectural Engineering, Henan University of Science and Technology, Luoyang 471003, China
[b] Department of Mechanical Engineering, Kyushu Institute of Technology, Kitakyushu-city 804-8550, Japan

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Abstract

Two types of optimization of thin-walled cylindrical shells loaded by lateral pressure are analyzed in this paper, with arbitrary axisymmetric boundary conditions and the volume being constant. The first is to find the optimal thickness to minimize the maximum deflection of a cylindrical shell. Here expressions of the objective function are obtained by the stepped reduction method. The optimal designs are reduced to nonlinear programming problems with an equality constraint. In minimizing the maximum deflection, the position of the maximum deflection from a previous iteration is used as the next one. The second is to find the optimal thickness to maximize the buckling pressure of shell. A buckling criterion of a shell is derived on the basis of an energy principle. An optimization criterion is formulated as the maximum of the buckling pressure. Moreover, the space of allowable solutions is defined. This procedure converges quickly and numerical results show the effectiveness of the method. Several examples are provided to illustrate the methods.

Keywords: Cylindrical shells; Deflection; Buckling pressure; Energy criterion; Optimal design

1. Introduction

Shell structures have been used widely in spaceflight and aviation, vessel, and large storage structures, etc. Optimal designs with respect to thin shell structures are important in both theory and application. Many techniques have been used for optimal design of shells. Often, the work addresses optimal design on weight of shell structures under various load and constraint conditions. However, few works deal with dual problems. Gajewski and Zyczkowski published their survey paper on optimal structural design under stability constraints in 1988 [1]. Hyman presented an optimum design for the instability of cylindrical shells under lateral pressure in 1971 [2]. Rotter stated the new European standard and current research needs on shell structures in 1998 [3]. Chapelle investigated fundamental considerations for the finite element analysis of shell structures [4] while Araar studied buckling of cylindrical shells under external pressure for a new shape of self-stiffened shell [5]. Adali did the minimum sensitivity design of laminated shells under axial load and external pressure [6]. Sakamoto did his investigation of a practical method of structural optimization by genetic algorithms [7]. Optimal sizes of a ground-based horizontal cylindrical tank under strength and stability constraints were investigated by Magnucki [8] and here are offered related works such as [9–13].

It is important to study the rational form of a shell to resist deformation. First, an effective way of optimal design for a thin cylindrical elastic shell is presented, which can determine the thickness functions that cause the minimax deflection or minimum compliance of the shell, under the conditions that the volume is constant and the middle surface shape is defined. In these optimal problems, the explicit formulations of the objective function cannot be determined by traditional methods that lead to much computational difficulties. The deflection solution of cylindrical shells with variable thickness can be given by the stepped reduction method; further explicit expressions of the objective function can be obtained. The expressions are suitable for the arbitrary axisymmetric boundary conditions and radial compression. The optimal design of a cylindrical shell is reduced to a nonlinear programming problem with an equality constraint.

Another method for optimal design of a thin cylindrical shell is proposed, which can determine the thickness functions

* Corresponding author. Address: College of Architectural Engineering, Henan University of Science and Technology, Luoyang 471003, China. Tel./fax: +86 379 6423 1735.

E-mail addresses: liangbin@mail.haust.edu.cn (B. Liang), noda@mech.kyutech.ac.jp (N. Noda), zsf@mail.haust.edu.cn (S. Zhang).

[1] Tel./fax: +81 93 884 3124.

[2] Tel./fax: +86 379 6423 1735.

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that cause the maximum buckling load, under the condition of the volume being constant. By use of an energy principle the bifurcation buckling of the cylindrical shell subjected to lateral uniform pressure is analyzed. The necessary condition of the shell system being in a stable state is that the second variation of the total potential energy equals zero when it reaches a disturbed bifurcation buckling state from a stable equilibrium state. The solution for buckling pressure of the shell is transferred to a generalized eigenvalue equation. The buckling pressure is expected to be the maximum and then the optimal design of the shell is reduced to a nonlinear programming problem with constraints of the volume being constant.

2. Governing equation and the solution

2.1. Deflection of the cylindrical shell

Consider the thin cylindrical elastic shell shown in Fig. 1, with the axisymmetric variable thickness $h(x)$, length l, radius r, elastic constants E, v, and arbitrary axisymmetric radial compression $P(x)$. Divide the shell into n segments. Let each shell segment be short enough so that it can be considered as having uniform thickness and being subjected to a uniform lateral pressure. Suppose the ith segment has the length $l_i=1/n$, thickness h_i, lateral pressure P_i, local variable x_i, $0\le x_i\le l_i$ (lower section of the segment $x_i=0$ and the upper section $x_i=l_i$). Then the differential equation for the radial deflection w_i of the ith shell segment is [14]

$$\frac{d^4 w_i(x_i)}{dx_i^4}+4K_i w_i(x_i)=\frac{P_i}{D_i} \tag{1}$$

where $D_i=Eh_i^3/12(1-\nu^2)$ is the radial stiffness and $K_i=3(1-\nu^2)/(rh_i)^2$. The solution of Eq. (1) can be written by the stepped reduction method as follows

$$w_i(x_i)=w_i(0)F_{1i}(x_i)+w_i'(0)F_{2i}(x_i)+M_i(0)F_{3i}(x_i)+Q_i(0)F_{4i}(x_i)+F_{5i}(x_i) \tag{2}$$

where $w_i(0)$, $w_i'(0)$, $M_i(0)$ and $Q_i(0)$ are the deflection, slope, bending moment and shear force, respectively, at the point $x_i=0$, and

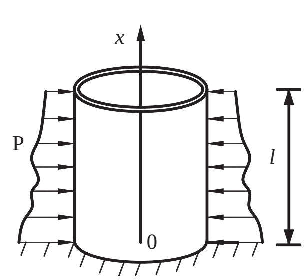

Fig. 1. Cylindrical shell with variable thickness.

$$\left.\begin{aligned}
F_{1i}(x_i)&=\cosh(\lambda_i x_i)\cos(\lambda_i x_i)\\
F_{2i}(x_i)&=[\cosh(\lambda_i x_i)\sin(\lambda_i x_i)+\sinh(\lambda_i x_i)\cos(\lambda_i x_i)]/(2\lambda_i)\\
F_{3i}(x_i)&=\sinh(\lambda_i x_i)\sin(\lambda_i x_i)/(2\lambda_i^2)\\
F_{4i}(x_i)&=[\cosh(\lambda_i x_i)\sin(\lambda_i x_i)-\sinh(\lambda_i x_i)\cos(\lambda_i x_i)]/(4\lambda_i^3)\\
F_{5i}(x_i)&=[1-F_{1i}(\lambda_i x_i)]P/(4\lambda_i D_i)
\end{aligned}\right\}$$

where $\lambda_i^4=Eh_i/4r^2D_i$. For convenience, the following dimensionless variables are introduced

$$\left.\begin{aligned}
&\bar{r}=\frac{r}{l},\ \bar{l}_i=\frac{l_i}{l},\ \bar{h}_i=\frac{h_i}{h_0},\ \bar{x}_i=\frac{x_i}{l},\ \bar{w}_i=\frac{w_i}{l},\ \bar{P}_i=\frac{P_i l^3}{D_0},\\
&\bar{M}_i=\frac{M_i l}{D_0},\ \bar{Q}_i=\frac{Q_i l^2}{D_0},\ D_0=\frac{Eh_0^3}{12(1-\nu^2)}
\end{aligned}\right\} \tag{3}$$

where h_0 denotes the thickness of a uniform shell with given volume V_0. Then Eq. (2) can be written

$$\bar{w}_i(x_i)=\bar{w}_i(0)f_{1i}(\bar{x}_i)+\bar{w}_i'(0)f_{2i}(\bar{x}_i)+\bar{M}_i(0)f_{3i}(\bar{x}_i)+\bar{Q}_i(0)f_{4i}(\bar{x}_i)+f_{5i}(\bar{x}_i) \tag{4}$$

where the quantities $\bar{w}_i(0)$, $\bar{w}_i'(0)$, $\bar{M}_i(0)$ and $\bar{Q}_i(0)$ denote dimensionless deflection, slope, bending moment, and shear force, respectively, at $\bar{x}_i=0$. The functions $f_{ki}(\bar{x}_i)$ are the corresponding dimensionless functions of $F_{ki}(x_i)$, $(k=1,2,3,4,5)$. Thus, $\bar{w}_i(\bar{x}_i)$ is an explicit formulation of $\bar{x}_i$. Let

$$S_i(\bar{x}_i)=[\bar{w}_i(\bar{x}_i),\bar{w}_i'(\bar{x}_i),\bar{M}_i(\bar{x}_i),\bar{Q}_i(\bar{x}_i)]^T \tag{5}$$

According to the relations of deflection, slope, bending moment and shear force, we have

$$S_i(\bar{x}_i)=T_i(\bar{x}_i)S_i(0)+U_i(\bar{x}_i)\quad (i=1,2,\ldots,n) \tag{6}$$

where $T_i(\bar{x}_i)$ and $U_i(\bar{x}_i)$ denote a 4×4 matrix and a 4×1 matrix, respectively, which consist of $f_{ki}(\bar{x}_i)$, $(k=1,2,3,4,5)$ and their derivatives. Continuity conditions at the junction of two neighbouring segments must be satisfied, so

$$S_i(0)=S_{i-1}(\bar{l}_{i-1})\quad (i=2,3,\ldots,n) \tag{7}$$

Thus, we have

$$\left.\begin{aligned}
&S_1(\bar{x}_1)=T_1(\bar{x}_1)S_1(0)+U_1(\bar{x}_1)\\
&S_i(\bar{x}_i)=T_i(\bar{x}_i)S_1(0)\prod T_j(\bar{l}_j)+U_i(\bar{x}_i)+T_i(\bar{x}_i)\left[U_{i-1}\left(\bar{l}_{i-1}\right)\right.\\
&\qquad \left.+\sum U_j\left(\bar{l}_j\right)\prod T_k\left(\bar{l}_k\right)\right]\quad (i=2,3,\ldots,n),\\
&\qquad (j=1,2,\ldots,i-2),\ (k=1,2,\ldots,j-1)
\end{aligned}\right\} \tag{8}$$

Then $T_i(\bar{x}_i)$ are the explicit formulation of $\bar{x}_i$, $\bar{h}_i$. When $i=n$ and $\bar{x}_n=\bar{l}_n$, Eq. (8) will give expressions for $\bar{w}_n$, $\bar{w}_n'$, $\bar{m}_n$ and $\bar{q}_n$. If the boundary conditions of the shell are given, the explicit expressions of the deflection, slope, bending moment and shear force at any point of the shell can be obtained from Eq. (8).

2.2. Buckling of cylindrical shell with variable thickness

Consider the same thin elastic cylindrical shell shown in Fig. 1, but under uniform lateral pressure P. Divide the shell into n elements along the x direction.

To study the stability of the cylindrical shell under lateral pressure, suppose that it is in a stable equilibrium state e, and is then given a disturbance to another state f. Let the displacement field in state f be

$$\mathbf{V}^f = \mathbf{V}^e + \mathbf{V}^j \tag{9}$$

where $\mathbf{V}^e$ is the displacement field in state e, and $\mathbf{V}^j$ is the disturbing displacement field from state e to f. The displacement can be expressed as linear functions of a shell element on the generalized joints [13].

$$\left.\begin{aligned} u^e &= B_1(x)u_1^e + B_2(x)u_2^e \\ w^e &= A_1(x)w_1^e + A_2(x)\theta_1^e + A_3(x)w_2^e + A_4(x)\theta_2^e \\ u^j &= [B_1(x)u_1^j + B_2(x)u_2^j]\cos j\theta \\ v^j &= [B_1(x)v_1^j + B_2(x)v_2^j]\sin j\theta \\ w^j &= \left[A_1(x)w_1^j + A_2(x)\theta_1^j + A_3(x)w_2^j + A_4(x)\theta_2^j\right]\cos j\theta \end{aligned}\right\} \tag{10}$$

where $(u_i^e, w_i^e, \theta_i^e)$, $(u_i^j, v_i^j, w_i^j, \theta_i^j)$, $(i=1,2)$ denote the generalized joint displacements in state e and the disturbing procedure, respectively, j is circumferential integral wave number, $A_i(x)$, $(i=1,2,3,4)$ and $B_i(x)$, $(i=1,2)$ are polynomials.

According to the relation between strain and displacement from Donnell theory, by substitution from the displacement field (u^e+u^j, v^j, w^e+w^j) into the disturbing state f, the strain and curvature change in state f can be denoted

$$\left.\begin{aligned} \varepsilon^f &= \varepsilon^e + \varepsilon^{(1)} + \varepsilon^{(2)} \\ \chi^f &= \chi^e + \chi^{(1)} \end{aligned}\right\} \tag{11}$$

where ε^e, χ^e are strain and curvature change, respectively, in state e, $\varepsilon^{(1)}, \chi^{(1)}$ and $\varepsilon^{(2)}$ are linear and quadratic terms, respectively, with u^j, v^j, w^j. The strain energy of the cylindrical shell in state f is denoted

$$U^f = U^e + U^{(1)} + U^{(2)} + 0(U^{(3)}) \tag{12}$$

where U^e is strain energy in state e, $U^{(1)}$, $U^{(2)}$ are linear and quadratic terms, respectively, with u^j, v^j, w^j.

Consider a virtual displacement in state f. From the principle of virtual work

$$\delta(U^f) = \iint (\mathbf{P}\cdot\delta\mathbf{V}) r \, d\theta \, dx \tag{13}$$

This can be rewritten as

$$\delta(U^f + \Omega^f) = 0 \tag{14}$$

where Ω^f is the external force potential energy in state f. Retaining the first-order and second-order terms and neglecting higher order terms, the following formula is derived from Eq. (14)

$$\delta(U^e + \Omega^e + U^{(1)} + \Omega^{(1)} + U^{(2)} + \Omega^{(2)}) = 0 \tag{15}$$

where $\Omega^{(1)} + \Omega^{(2)}$ is the increment of external force potential energy from state e to f. Because state e is an equilibrium state, the following equilibrium equation is obtained from the principle of minimum potential energy

$$\delta(U^e + \Omega^e) = 0 \tag{16}$$

Meanwhile state f is also an equilibrium state. So the first-order variation of total potential energy in state f equals zero

$$\delta(U^{(1)} + \Omega^{(1)}) = 0 \tag{17}$$

From Eqs. (15), (16) and (17)

$$\delta(U^{(2)} + \Omega^{(2)}) = 0 \tag{18}$$

Eq. (18) shows that the second order variation of total potential energy from state e to f is zero. When the criterion is satisfied, unique solution of the disturbing generalized joint displacement will not exist. Then bifurcation instability appears.

In the disturbing procedure from state e to f, the second-order term for the external force work is

$$\begin{aligned} \Omega^{(2)} = \frac{P}{2} \iint \Bigg[& (w^j)^2 + w^j \left(\frac{\partial v^j}{\partial \theta}\right) + r w^j \left(\frac{\partial u^j}{\partial x}\right) + (v^j)^2 \\ & - v^j \left(\frac{\partial w^j}{\partial \theta}\right) - r u^j \left(\frac{\partial w^j}{\partial x}\right) \Bigg] d\theta \, dx \end{aligned} \tag{19}$$

According to the continuity condition at the joints of neighboring shell elements, Eq. (8) becomes

$$[K^e]\{q^e\} = \{Q^e\} \tag{20}$$

where $[K^e]$ is the rigidity matrix, $\{q^e\}$, $\{Q^e\}$ are the generalized joint displacement and force, respectively. The equilibrium displacement $\{q^e\}$ can be derived from Eq. (20) under appropriate boundary conditions. A matrix expression of the disturbing generalized joint displacement is obtained by analyzing terms of the criterion Eq. (18)

$$\{[K^j] + P[K^g]\}\{q^j\} = \{0\} \tag{21}$$

where $[K^j]$ and $[K^g]$ are rigidity matrix and increment rigidity matrix with j circumferential wave number. $\{q^j\}$ is the disturbing generalized joint displacement. Eq. (21) is a generalized eigen equation derived by the stability criterion, from which the buckling load can be obtained.

For convenience of numerical computation, the following dimensionless variables are introduced

$$\bar{r} = \frac{r}{l}, \bar{l}_i = \frac{l_i}{l}, \bar{h}_i = \frac{h_i}{h_0}, \bar{x}_i = \frac{x_i}{l}, \bar{w}_i = \frac{w_i}{l}, \{\bar{q}\} = \frac{\{q\}}{l}, \bar{P} = \frac{r^3 P}{D_0}, D_0 = \frac{E h_0^3}{12(1-\nu^2)}$$

where h_0 is the average thickness of a shell with the given volume. Then dimensionless Eqs. (20) and (21) are

$$[\bar{K}^e]\{\bar{q}^e\} = \{\bar{Q}^e\} \quad (22)$$

$$\{[\bar{K}^j] + \bar{P}[\bar{K}^g]\}\{\bar{q}^j\} = \{0\} \quad (23)$$

where $[\bar{K}^e]$ and $\{\bar{Q}^e\}$ are the dimensionless global rigidity matrix and the generalized joint force of Eq. (22), respectively. $[\bar{K}^j]$ and $[\bar{K}^g]$ are the dimensionless global rigidity matrix and the increment rigidity matrix of Eq. (23), respectively.

The generalized joint displacement $\{\bar{q}^e\}$ can be obtained from Eq. (22) under appropriate boundary conditions, then the minimum eigenvalue of Eq. (23) can be obtained by using the inverse iteration method. Here, the buckling pressure is the minimum eigenvalue for various circumferential wave number j

$$\bar{p}_{\rm cr} = {\rm Min}\{\bar{p}_{cr}(j), \quad (j = 1,2,\ldots,10)\} \quad (24)$$

3. Two types of optimal design

3.1. Optimal design on minimax deflection

The following procedure is proposed to determine the optimal design. Determine the thickness function $h(x)$, which minimizes the maximal $w(h,x)$ subject to the constraint of its volume being constant $\int h(x){\rm d}x = V_0/2\pi r$. As previously stated, after dividing the shell into n shell segments and setting a dimensionless transformation, the optimal design problem can be defined as follows. Determine $n+1$ variables $(\bar{h}_1,\bar{h}_2,\ldots,\bar{h}_n,\bar{x})$,

$$\left.\begin{array}{ll}\text{Minimize} & \max \bar{w}(\bar{h}_1,\bar{h}_2,\ldots,\bar{h}_n,\bar{x}),\\ \text{subject to} & \sum \bar{h}_i\bar{l}_i = 1,(\bar{h}_i \geq \bar{h}_{\min})(i = 1,2,\ldots,n)\end{array}\right\} \quad (25)$$

where $\bar{h}_{\min}$ is the dimensionless given minimal thickness.

If the boundary conditions and $\bar{h}_i$ $(i = 1,2,\ldots,n)$ are given, the solution $\bar{w}$ can be obtained by Eq. (8). To find the point of the maximal deflection, let the length of each segment be small enough so that only one stationary point of deflection exists at each segment, if it exists. The following procedure is proposed to determine the stationary point of deflection. First, determine all segments $\{j\}$, which satisfy $\bar{w}_j \cdot \bar{w}_{j-1} \leq 0$. By means of the average section method, the stationary points of deflection at these elements will be obtained. Denote these points by $\{\bar{x}_j\}$. Second, compare the deflections corresponding to points $\{\bar{x}_j\}$, and find the point of maximal deflection for the whole shell. Denote the point as $\bar{x}_m$, and the deflection as $\bar{w}_m$, $\bar{w}_m = \bar{w}_m(\bar{h}_1,\bar{h}_2,\ldots,\bar{h}_n,\bar{x}_m)$, where m indicates the point $\bar{x}_m$ lying in the mth segment. Thus, the optimization objective is to determine $\bar{h}_1,\bar{h}_2,\ldots,\bar{h}_n$, to minimize $\bar{w}_m(\bar{h}_1,\bar{h}_2,\ldots,\bar{h}_n,\bar{x}_m)$. Model (25) is a nonlinear programming problem with an equality constraint, and the explicit formulation of the objective function with respect to design variables was obtained. Therefore, the derivatives of the objective function with respect to design variables can be easily obtained. It will be solved by using various multidimensional gradient methods. For example, if the reduced gradient method is used and $\bar{h}_n$ is taken as the base variable, the reduced gradient can be easily obtained as follows:

$$\left[\frac{\partial w_m(h_1,\ldots,h_{n-1},x_m)}{\partial h_1},\ldots,\frac{\partial w_m(h_1,\ldots,h_{n-1},x_m)}{\partial h_{n-1}}\right].$$

As a nonlinear programming with an equality constraint, Eq. (25) can be solved by the simplified projection gradient method. Because the gradient vector is obtained easily, the proposed method with high calculation accuracy and fast convergence is superior to most traditional methods.

3.2. Optimal design on maximizing buckling pressure

The optimal design of a cylindrical shell on stability can be stated as follows. Dividing the shell into n shell segments and setting a dimensionless transformation, and keeping the shell volume constant, find the optimal thickness $\bar{h}_i$, $(i = 1,2,\ldots,n)$ to maximize its buckling pressure

$$\left.\begin{array}{ll}\text{minimize} & F = -\bar{P}_{\rm cr}(\bar{h}_1,\bar{h}_2,\ldots,\bar{h}_n)\\ \text{subject to} & \sum \bar{h}_i\bar{l}_i = 1,(\bar{h}_{\max} \geq \bar{h}_i \geq \bar{h}_{\min}), \quad (i = 1,2,\ldots,n)\end{array}\right\} \quad (26)$$

where $\bar{h}_{\max}$, $\bar{h}_{\min}$ are the given dimensionless maximum and minimum thickness, respectively. The following procedure is proposed to determine the optimal design. The Tabu search algorithm is used for pilot calculations for a representative material. The results show high calculation accuracy but slow convergence for the tabu search algorithm.

Being a nonlinear programming with an equality constraint, (26) can be solved by the simplified projection gradient method with high calculation accuracy and fast convergence.

4. Numerical results and discussion

4.1. Numerical results on minimax deflection

In this section, some numerical results for the two types of optimal design obtained using the stepped reduction method and suitable optimal techniques are illustrated. Several typical examples are provided. In the following numerical examples, let initial parameters: $\bar{h}_i^{(0)} = 1$, $(i = 1,2,\ldots,n)$, $\nu = 0.3$, $\bar{h}_0/\bar{r} = 0.01$, $\bar{l}/\bar{r} = 2.0$. λ denotes the ratio of maximal deflection of the optimal shell and maximal deflection of the uniform shell with the same volume. In Fig. 2, the solid lines represent the thickness distribution and deflection curves of the optimal shell, respectively. Meanwhile, the dotted lines show the corresponding thickness distribution and deflection curves of the uniform shell with the same volume, respectively. (a) Is both ends built-in, uniform load, $\lambda = 0.760$; (b) is both ends simply supported, uniform load, $\lambda = 0.798$; (c) is lower end built-in, upper end free, and $P(x) = P(1-x/l)$, $\lambda = 0.619$.

4.2. Numerical results on maximizing buckling pressure

It is effective to calculate the buckling pressure of a cylindrical shell with variable thickness by using the proposed

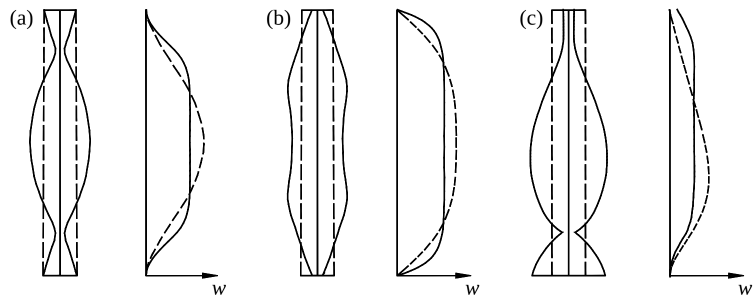

Fig. 2. Curves of optimal thickness and deflection under various boundary conditions.

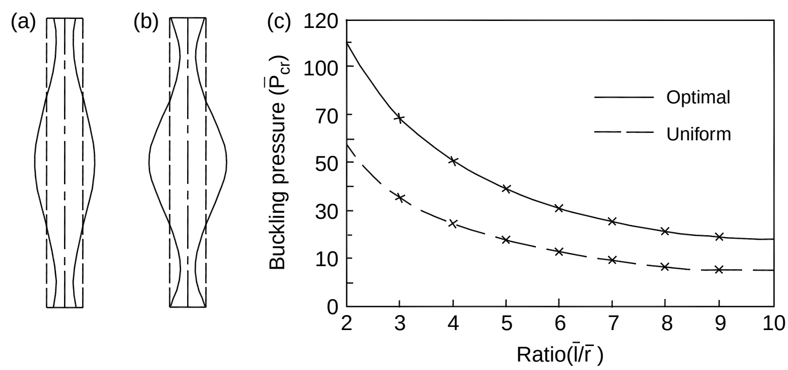

Fig. 3. Curves of optimal thickness and buckling pressure under C1 boundary conditions.

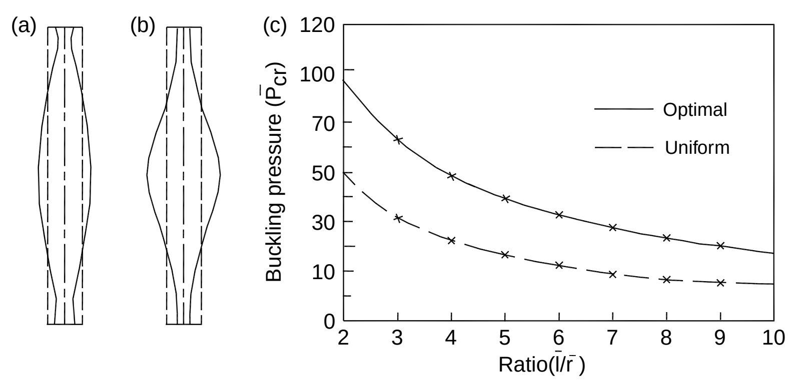

Fig. 4. Curves of optimal thickness and buckling pressure under S3 boundary conditions.

method. Numerical results for uniform thickness show that the results are close in comparison with that by Donnell theory. Some numerical results for various boundary conditions are obtained. Here, the optimal results for two typical boundary conditions are illustrated, with various ratios of radius to thickness, and length to radius. The boundary conditions are

$$\text{C1 boundary}: \ u = v = w = \frac{\partial w}{\partial x} = 0$$

$$\text{S3 boundary}: N_x = v = w = M_x = 0$$

Let the initial parameters $\bar{h}_i^{(0)} = 1$, $(i = 1,2,\ldots,n)$, $\nu = 0.3$, $\bar{h}_0/\bar{r} = 0.01$, $n = 160$, and $\lambda = \bar{P}^*_{cr}/\bar{P}_{cr}$, where $\bar{P}_{cr}$ and $\bar{P}^*_{cr}$ are the buckling pressure of the optimal shell and the uniform shell with the same volume, respectively. Results are shown in Figs. 3 and 4.

(a) $\bar{l}/\bar{r} = 2$, $\bar{P}_{cr} = 69.61$, $\bar{P}^*_{cr} = 121.35$, $\lambda = 1.74$.
(b) $\bar{l}/\bar{r} = 10$, $\bar{P}_{cr} = 12.96$, $\bar{P}^*_{cr} = 27.74$, $\lambda = 2.14$.
(c) buckling pressure as a function of $\bar{l}/\bar{r}$.
(a) $\bar{l}/\bar{r} = 2$, $\bar{P}_{cr} = 49.76$, $\bar{P}^*_{cr} = 90.77$, $\lambda = 1.82$.
(b) $\bar{l}/\bar{r} = 10$, $\bar{P}_{cr} = 8.03$, $\bar{P}^*_{cr} = 21.44$, $\lambda = 2.67$.
(c) buckling pressure as a function of $\bar{l}/\bar{r}$.

5. Conclusions

In this paper, we have investigated two types of optimization of cylindrical shells. First, the stepped reduction method has been used to calculate the deflection of a cylindrical shell; further explicit expressions of the objective function can be derived and analyzed by use of the iteration method. The expressions are suitable for arbitrary axisymmetric boundary conditions and radial compression. The optimal design of a cylindrical shell is reduced to a nonlinear programming problem with an equality constraint. Secondly, an effective method to maximize the buckling pressure of a cylindrical shell with variable thickness and subjected to lateral uniform pressure is proposed. The bifurcation buckling of the cylindrical shell has been derived by use of an energy principle. If it reaches a disturbing bifurcation buckling state from a stable equilibrium state, the necessary condition of the cylindrical shell being in a stable state is that the second variation of total potential energy equals zero. To solve the buckling pressure of the cylindrical shell is reduced to a generalized eigenvalue equation. Some numerical calculations are carried out to show the usefulness of the present method. The numerical examples demonstrate the accuracy and efficiency of the proposed method.

References

[1] Gajewski A, Zyczkowski M. Optimal structural design under stability constraints. The Netherlands: Kluwer Academic Publishers; 1988.

[2] Hyman BI, Lucas Jr AW. An optimum design for the instability of cylindrical shells under lateral pressure. AIAA J 1971;9:738–40.

[3] Rotter JM. Shell structures: the new European standard and current research needs. Thin-walled Struct 1998;31(1–3):3–23.

[4] Chapelle D, Bathe KJ. Fundamental considerations for the finite element analysis of shell structures. Comput Struct 1998;66(1):19–36.

[5] Araar M, Julien JF. Buckling of cylindrical shells under external pressure proposition of a new shape of self-stiffened shell. Struct Eng Mech 1996; 4(4):451–60.

[6] Adali S, Verijenko VE, Richter A. Minimum sensitivity design of laminated shells under axial load and external pressure. Compos Struct 2001;54(2–3):139–42.

[7] Sakamoto H, Takada S. Investigation of a practical method of structural optimization by genetic algorithms. JSME Int J Ser A (Solid Mech Struct Eng) 2001;44(3):330–7.

[8] Magnucki K, Lewinski J, Stasiewicz P. Optimal sizes of a ground-based horizontal cylindrical tank under strength and stability constraints. Int J Press Vessels Piping 2004;81(12):913–7.

[9] Rafig MY, Southcombe C. Genetic algorithms in optimal design and detailing of reinforced concrete biaxial columns supported by a declarative approach for capacity checking. Comput Struct 1998;69(4): 443–57.

[10] Frecker M, Kikuchi N, Kota S. Optimal design of compliant mechanisms, (in Warsaw, Poland) WCSMO-2, vol. 1; 1997. p. 345–350.

[11] Ory H, Reimerdes HG, Schmid T, Rittweger A, Garcia JG. Imperfection sensitivity of an orthotropic spherical shell under external pressure. Int J Non-Linear Mech 2002;37(4–5):669–86.

[12] Baker EH, Kovalevsky L, Rish FL. Structural analysis of shells. New York: McGraw Hill; 1972.

[13] Teng JG, Rotter JM. Buckling of thin metal shell. London/New York: Spon Press/Taylor & Francis Group; 2004.

[14] Timoshenko S, Woinowsky-Krieger S. Theory of plates and shells. New York: McGraw Hill; 1959.