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Accurate Determination of Stress Intensity Factor for Interface Crack by Finite Element Method

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Abstract. This paper presents the simple method to determine the complex stress intensity factor of interface crack problem by the finite element method. The proportional method is extended to the interface crack problem. In the present method, the stress values at the crack tip calculated by FEM are used and the stress intensity factors of interface crack are evaluated from the ratio of stress values between a given and a reference problems. A single interface crack in an infinite bi-material plate subjected to tension and shear is selected as the reference problem in this study. The accuracy of the present analysis is discussed through the results obtained by other methods. As the result, it is confirmed that the present method is useful for analyzing the interface crack problem.

Introduction

In recent years, use of composite materials has been increasing in wide engineering field and accurate evaluation of interface strength in dissimilar materials has become important. Many methods have been developed to calculate the stress intensity factors of an interface crack in dissimilar materials by using the finite element method (FEM) [1]. However, it is still not necessarily easy to analyze the stress intensity factors of interface crack by FEM because of the oscillatory stress singularity.

In the crack problem in homogeneous material, Kisu et al [2] have proposed the simple method using the one point stress value near the crack tip. This method is called the proportional method and is based on the fact that the stress distribution near the crack tip is proportional to $1/\sqrt{r}$. Recently, Nisitani et al [3, 4] have developed the crack tip stress method for calculating the highly accurate stress intensity factors by FEM The method is used only the stress values at the nodal point of crack tip analyzed by FEM Although the stress value at the crack tip calculated by FEM contain numerical error, the value is effective as a measure of the magnitude of singular stress field. In the interface crack problem, however, the highly accurate values of the stress intensity factors cannot be evaluated by using the crack tip stress values.

This paper presents the simple method to calculate the stress intensity factors for interface crack by FEM. The proportional method is extended to the interface crack problem. The accuracy of the present analysis is verified through the comparing the present results with the results obtained by other researches. The calculation shows that the present method has the sufficient accuracy in the case of interface crack problems.

Method of Analysis

Principle of proportional method. In this section, the principle of the proportional method is explained by taking a two dimensional mode I crack problem as an example.

In the fundamental concept of linear fracture mechanics, stress distribution near the crack tip is expressed by following equation when $\theta = 0$.

$$\sigma_y = K_I / \sqrt{2\pi r} \quad (1)$$

When the distance from crack tip r= r_0, K_I / σ_y is constant and the next relation between two different problems can be obtained theoretically.

$$K_I * / \sigma_y * = K_I / \sigma_y \quad (2)$$

If the stress intensity factor K_1* of the reference problem is known, we can easily obtained the K_1 of given problem from the relation (2) because the stresses σ_y * and σ_y of reference and given problems can be calculated by FEM.

In the interface crack problem, however, the highly accurate values of the stress intensity factors cannot be obtained by using the stress values near the interface crack tip because the elastic solution of the interface crack has an oscillatory stress singularity.

Application of proportional method to interface crack problem. In this study, the proportional method is extended to interface crack problem. In the ordinary extrapolation method of interface crack problem, the stress intensity factors are evaluated by the following equations.

$$K_1 = \lim_{r \to 0} \sqrt{2\pi r} \sigma_y \left(\cos Q + \frac{\tau_{xy}}{\sigma_y} \sin Q \right), \; K_2 = \lim_{r \to 0} \sqrt{2\pi r} \tau_{xy} \left(\tau_{xy} \cos Q - \frac{\sigma_y}{\tau_{xy}} \sin Q \right) \quad (3)$$

$$Q = \varepsilon \ln\left(\frac{r}{2a} \right), \; \varepsilon = \frac{1}{2\pi} \ln\left[\left(\frac{\kappa_1}{G_1} + \frac{1}{G_2}\right) / \left(\frac{\kappa_1}{G_1} + \frac{1}{G_2}\right) \right],$$

$$\kappa_j = (3 - \nu_j)/(1 + \nu_j) \text{ Plane stress, } \kappa_j = 3 - 4\nu_j \text{ Plane strain (j=1, 2)} \quad (4)$$

where G_j and ν_j are shear modulus and Poisson's ratio of material j (j=1, 2) and ε is oscillation index. If $\varepsilon = \varepsilon *$ and $\tau_{xy} / \sigma_y = \tau_{xy} * / \sigma_y *$ at r=r_0, the oscillatory terms in Eq.(3) are the same between two different problems. Then, Eq. (5) is obtained, as well as homogeneous crack problem.

$$K_1 * / \sigma_y * = K_1 / \sigma_y , \quad K_2 * / \tau_{xy} * = K_2 / \tau_{xy} \quad (5)$$

Eq. (5) is based on the similar stress distributions near the interface crack tip and the asterisk * means the reference problem. In FEM analysis, we use the next condition in order to obtain the similar stress distributions near the interface crack.

$$\frac{\tau_{xy,FEM} *}{\sigma_{y,FEM} *} = \frac{\tau_{xy,FEM}}{\sigma_{y,FEM}} \quad (6)$$

Here, the values of $\sigma_{y,FEM}$ *, $\tau_{xy,FEM}$ * and $\sigma_{y,FEM}$, $\tau_{xy,FEM}$ are the stress values calculated by FEM. It is noted that the same analysis conditions, that is the same material constants, the same crack length and the same mesh pattern near crack tip, have to be used in the calculation of $\sigma_{y,FEM}$ *, $\tau_{xy,FEM}$ * and $\sigma_{y,FEM}$, $\tau_{xy,FEM}$. Therefore, the SIFs of the given problem can be easily evaluated from

$$K_1 \fallingdotseq \frac{\sigma_{y,FEM}}{\sigma_{y,FEM} *} K_1 * , \quad K_2 \fallingdotseq \frac{\tau_{xy,FEM}}{\tau_{xy,FEM} *} K_2 * \quad (7)$$

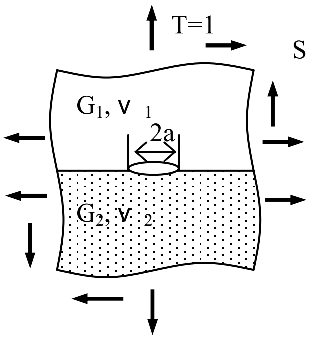


Fig.1 Reference problem (K_1*, K_2* are known)

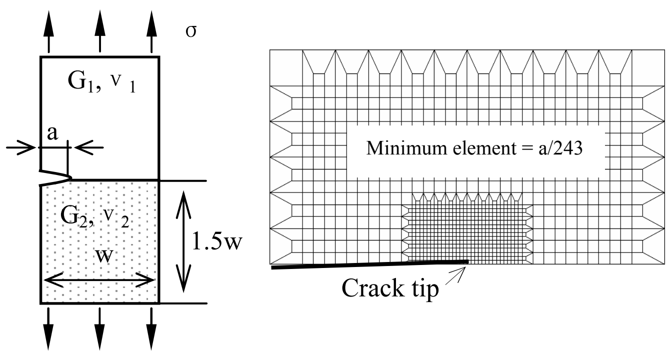


Fig.2 Given problem and FEM mesh pattern

In this study, K_1* and K_2* of a reference problem (Fig.1) are given by the exact solution of single interface crack in an infinite dissimilar plate subjected to tension T and shear S.

$$K_1 * + iK_2 * = (T + iS)\sqrt{\pi a}(1 + 2i\varepsilon) \tag{8}$$

The loading stresses T and S in reference problem as shown in Fig.1 are determined from the Eq.(6).

Determination of loading stresses. As reference problem, an interface crack in an infinite bi-material plate subjected to tension and shear loads (Fig.1) is used in this study. Actually, this plate is a square bi-material plate with an interface crack whose width is 1500 times of the crack size. In order to determine the T and S, a reference problem is expressed by superposing tension and shear problems. The stresses near interface crack tip in bonded dissimilar materials subjected to the loads T and S are expressed by

$$\sigma_{y,FEM} * = \sigma_{y,FEM}^{T=1} * \cdot T + \sigma_{y,FEM}^{S=1} * \cdot S\ ,\ \tau_{xy,FEM} * = \tau_{xy,FEM}^{T=1} * \cdot T + \tau_{xy,FEM}^{S=1} * \cdot S \tag{9}$$

where $\sigma_{y,FEM}^{T=1}$ is the stress σ_y of a tension problem analyzed by FEM when T=1 and S=0.

By substituting the expressions (9) into Eq.(6), we obtain the loading stress S when T=1.

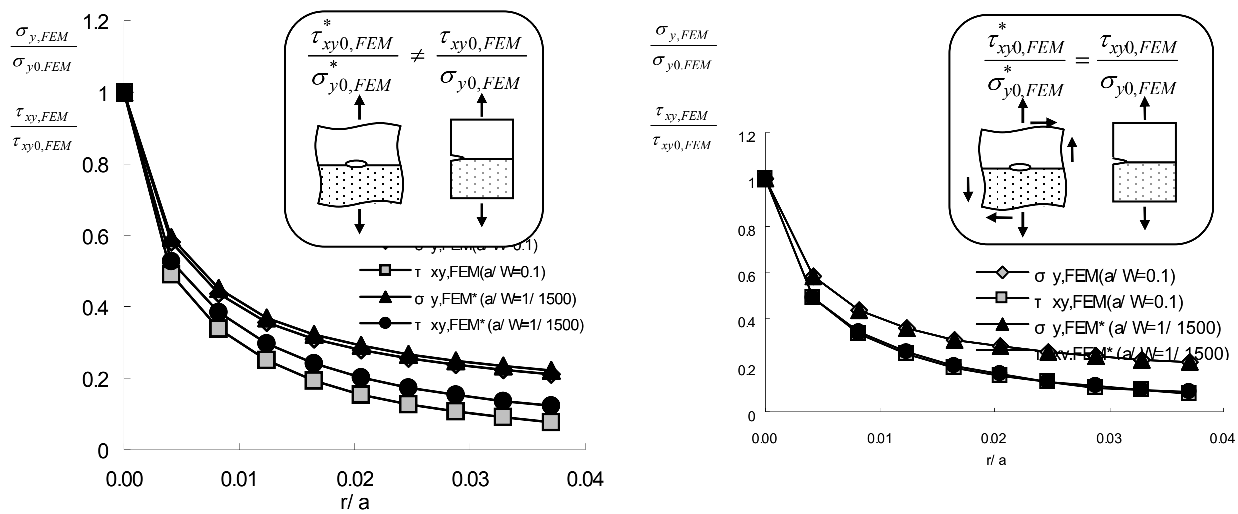
$$\frac{S}{T} = \frac{\sigma_{y,FEM} \cdot \tau_{xy,FEM}^{T=1} * - \tau_{xy,FEM}^{S=1} * \cdot \sigma_{y,FEM}^{T=1} *}{\tau_{xy,FEM} \cdot \sigma_{y,FEM}^{T=1} * - \sigma_{y,FEM} \cdot \tau_{xy,FEM}^{S=1} *} \tag{10}$$

The stress intensity factors of given problem are calculated by Eqs. (7), (8) and (10).

Results and discussions

The finite dissimilar plate with a single edge interface crack is analyzed to examine the usefulness of the present method. Fig.2 shows the problem of the finite bonded plate with edge interface crack and FEM mesh pattern. The minimum element size at the crack tip is a/243. The elements near the crack tip are made fine systematically. It should be noted that the same mesh patterns near the crack tip have to be used in the calculation of stress values for the given and reference problems.

Fig.3 shows the relative stress distributions near the interface crack tip. Here, $\sigma_{y0,FEM}$ and $\tau_{xy0,FEM}$ are stress values at the crack tip node obtained by FEM, and $\sigma_{y,FEM}$ and $\tau_{xy,FEM}$ are at any nodes. In these cases, the ratio G_2/G_1 is 10, and the relative crack length a/w is 0.1. As shown in these figures, when the condition (6) is satisfied, the relative stress distributions of given and reference problems coincide with each other. From this figure, it is seen that the singular stress field of the interface crack is controlled by $\tau_{xy0,FEM} / \sigma_{xy0,FEM}$ at the crack tip calculated by FEM.



(a) Eq. (6) is not satisfied. (b) Eq. (6) is satisfied.

Fig.3 Relative stress distributions near interface crack tip.

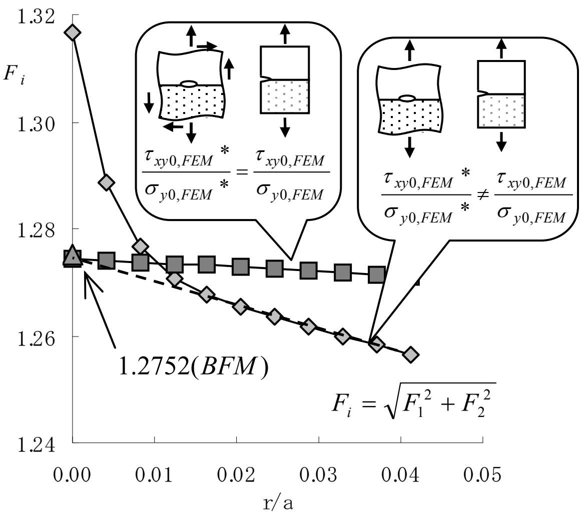


Fig. 4 F_i-values calculated by the present method.

Table 1 Normalized stress intensity factors of Fig.1. ($\nu_1 = \nu_2 = 0.3$, Plane stress)

G_2/G_1	a/w	$F_1 = K_1/(\sigma\sqrt{\pi a})$			$F_2 = K_2/(\sigma\sqrt{\pi a})$		
		Present	Ref.(5)	Ref.(6)	Present	Ref.(5)	Ref.(6)
4.0	0.1	1.207	1.201	1.209	-0.240	-0.238	-0.238
	0.2	1.365	1.387	1.368	-0.251	-0.254	-0.250
	0.3	1.644	1.653	1.654	-0.286	-0.288	-0.288
	0.4	2.092	2.100	2.100	-0.359	-0.359	-0.358
	0.5	2.791	2.807	2.806	-0.484	-0.483	-0.483
10.0	0.1	1.228	1.220	1.229	-0.341	-0.338	-0.338
	0.2	1.367	1.367	1.369	-0.350	-0.349	-0.348
	0.3	1.643	1.646	1.648	-0.400	-0.398	-0.398
	0.4	2.082	2.088	2.089	-0.495	-0.495	-0.493
	0.5	2.772	2.788	2.787	-0.663	-0.664	-0.661

Fig.4 shows the stress intensity factor F_i obtained by present method. The dimensionless value F_i is defined by

$$F_i = \sqrt{F_1^2 + F_2^2} = \sqrt{K_1^2 + K_2^2}\,/(\sigma\sqrt{\pi a}) \tag{11}$$

The exact value calculated by the body force method is 1.2752 [4] in this case. As shown in Fig.4, when the condition (6) is satisfied, values of F_i obtained by present method are very close to 1.2752 at the crack tip node. However, F_i –values have a large error near the crack tip when the condition (6) is not satisfied. Therefore, in an interface crack problem, the stress value at the crack tip node is very effective as a measure of the strength of singular stress field, as well as a ordinary crack problem in homogeneous material [3].

Table 1 shows the normalized stress intensity factors of the single edge interface crack problem, as shown in Fig.2. The results are compared with other results analyzed by the boundary element method [5, 6]. The analyses are performed by changing the ratio of shear modulus G_2/G_1 and the relative crack length a/w. As seen from Table 1, the present results coincide with other results. Therefore, it can be said that the present method using FEM has sufficient accuracy in the case of interface crack problem.

References

[1] S.N. Atluri: Structural Integrity and Durability, Tech Science Press, Forsyth, GA (1997)

[2] H. Kisu, R. Yuuki and H. Kitagawa: Transactions of the Japan Society of Mechanical Engineers (in Japanese), Vol. 51-463, Series A (1985), p. 660

[3] H. Nisitani, T. Kawashima, W. Fujisaki and T. Fukuda: Transactions of the Japan Society of Mechanical Engineers (in Japanese), Vol. 65-629, Series A (1999), p. 26

[4] H. Nisitani and T. Teranishi: Key Engineering Materials, Vol. 243-244, (2003), p. 387

[5] R. Yuuki and S.B. Cho: Engineering Fracture Mechanics, Vol. 34, (1989), p. 179

[6] N. Miyazaki, T. Ikeda, T. Soda and T. Munakata: Engineering Fracture Mechanics, Vol. 45, No.5, (1993), p. 599