Key Engineering Materials Vols. 462-463 (2011) pp 1146-1151
Online available since 2011/Jan/20 at www.scientific.net

doi:10.4028/www.scientific.net/KEM.462-463.1146

Stress Intensity Factor of a Central Interface Crack in a Bonded Strip under Arbitrary Material Combination

Nao-Aki NODA [1, a], Yu ZHANG [1,b], Xin LAN [1,c], Ken-Taro TAKAISHI [1,d]
[1]Department of Mechanical Engineering, Kyushu Institute of Technology, 1-1 Sensui-cho, Tobata-ku, Kitakyushu-shi, Fukuoka, 804-8550 Japan
[a]noda@mech.kyutech.ac.jp, [b]zyzhangyu1225@yahoo.com, [c]xinlan_al@yahoo.com
[d]f104066k@tobata.isc.kyutech.ac.jp

Keywords: Elasticity, Stress Intensity Factor, Fracture Mechanics, Finite Element Method, Interface Crack, Bonded Strip

Abstract. Although a lot of interface crack problems were previously treated, few solutions are available under arbitrary material combination. This paper deals with one central interface crack and numerical interface cracks in a bonded strip. Then, the effects of material combination on the stress intensity factors are discussed. A useful method to calculate the stress intensity factor of interface crack is presented with focusing on the stress at the crack tip calculated by the finite element method. For one central interface crack, it is found that the results of bonded strip under remote uni-axial tension are always depending on the Dunders' parameters α , β and different from the well-known solution of the central interface crack under internal pressure that is only depending on β . Besides, it is shown that the stress intensity factor of bonded strip can be estimated from the stress of crack tip in the bonded plate when there is no crack. It is also found that $F_I < 1$ when $(\alpha+2\beta)(\alpha-2\beta) > 0$, $F_I > 1$ when $(\alpha+2\beta)(\alpha-2\beta) < 0$, and $F_I = 1$ when $(\alpha+2\beta)(\alpha-2\beta) = 0$. For numerical interface cracks , values of F_I and F_{II} with arbitrary material combination expressed by α , β are obtained.

Introduction

A central interface crack in a bonded plate has been treated in the previous studies [1,2,3], and the dimensionless stress intensity factors are provided in Table 1. As can been seen from this table, those results almost coincide with each other. However, the limiting solution as $a/W \to 0$ in Table 1 has not been discussed yet in the previous studies. In Table 1 it is seen that dimensionless stress intensity factor F_I does not approach unity although $F_{II} \to 0$ as $a/W \to 0$. In this study, stresses at the interface crack tip will be calculated by applying the finite element method. Then the stress intensity factors are determined from the results of the reference problem and given unknown problem [1] using the same finite element mesh pattern. Here, the most fundamental central interface crack in bonded plate in Fig. 1 will be considered with varying Dundur's parameter α, β . Then, the effects of material combination on the interface stress intensity factors K_I, K_{II} will be discussed.

Table 1 Dimensionless stress intensity factors of center interface crack in bonded plate (see Fig 1, Plane stress, $\nu_1 = \nu_2 = 0.3$). $K_I + iK_{II} = (F_I + iF_{II})(1+2i\varepsilon)\sigma\sqrt{\pi a}$

G_2/G_1	a/W	F_I			F_{II}		
		Ref. [1]	Ref. [2]	Ref. [3]	Ref. [1]	Ref. [2]	Ref. [3]
4	→0	?	?	?	?	?	?
	0. 1	0. 986	0. 981	0. 986	0. 005	0. 005	0. 005
	0. 2	1. 005	1. 005	1. 005	0. 009	0. 010	0. 009
100	→0	?	?	?	?	?	?
	0. 1	0. 943	0. 941	0. 943	0. 008	0. 008	0. 009
	0. 2	0. 960	0. 960	0. 960	0. 017	0. 018	0. 018

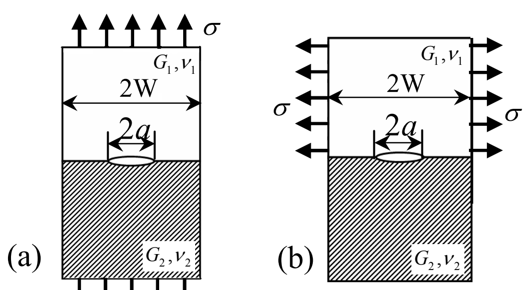


Fig.1 Bonded finite plate with a central interface crack .

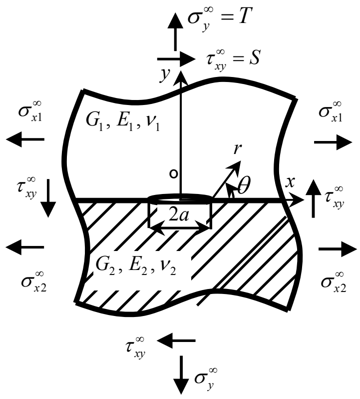


Fig.2 Reference problem ($\varepsilon_{x1} = \varepsilon_{x2}$ at $y = 0$)

Analysis Method

The analysis method used in this research is based on the stresses at the crack tip calculated by FEM. By using the proportional stress fields for the reference and given problems, stress intensity factors can be obtained with a good accuracy [4].

An effective method was recently proposed by Oda et al [1]successfully to analyze interface crack problems. It is well known that there exists oscillation singularity at the interface crack tip. From the stresses σ_y, τ_{xy} along the interface crack tip, stress intensity factors are defined as shown in Eq.(1).

$$\sigma_y + i\tau_{xy} = \frac{K_I + iK_{II}}{\sqrt{2\pi r}}\left(\frac{r}{2a}\right)^{i\varepsilon} \quad , \quad r \to 0, \tag{1}$$

$$\varepsilon = \frac{1}{2\pi}\ln\left[\left(\frac{\kappa_1}{G_1} + \frac{1}{G_2}\right) / \left(\frac{\kappa_2}{G_2} + \frac{1}{G_1}\right)\right] \tag{2}$$

$$\kappa_j = \frac{3-\nu_j}{1+\nu_j}(plane \quad stress), \kappa_j = 3 - 4\nu_j\ (plane \quad strain) \quad (j = 1,2)$$

From Eq.1, the stress intensity factors may be separated as

$$K_I = \lim_{r\to 0}\sqrt{2\pi r}\sigma_y\left(\cos Q + \frac{\tau_{xy}}{\sigma_y}\sin Q\right), \tag{3}$$

$$K_{II} = \lim_{r\to 0}\sqrt{2\pi r}\tau_{xy}\left(\cos Q + \frac{\sigma_y}{\tau_{xy}}\sin Q\right), \tag{4}$$

$$Q = \varepsilon \ln(\frac{r}{2a}) . \tag{5}$$

Here, r and Q can be chosen as constant values since the reference and unknown problem have the same mesh pattern and material combination. Therefore if Eq.(6) is satisfied, Eq.(7)may be derived from Eq.(6). In such case, oscillatory items of the reference and unknown problems are changed into the same.

$$Q^* = Q, \quad \frac{\tau_{xy}{}^*}{\sigma_y{}^*} = \frac{\tau_{xy}}{\sigma_y} \tag{6}$$

$$\frac{K_I{}^*}{\sigma_y{}^*} = \frac{K_I}{\sigma_y}, \quad \frac{K_{II}{}^*}{\tau_{xy}{}^*} = \frac{K_{II}}{\tau_{xy}} \tag{7}$$

Stress intensity factors of the given unknown problem can be obtained by:

$$K_I = \frac{\sigma_{y0,FEM}}{\sigma_{y0,FEM}{}^*}K_I{}^* \tag{8}$$

$$K_I = \frac{\sigma_{y0,FEM}}{\sigma_{y0,FEM}{}^*}K_I{}^* \tag{9}$$

Here, σ_y^*, τ_{xy}^* are stresses of reference problem calculated by FEM, and σ_y, τ_{xy} are stresses of given unknown problem. Stress intensity factors of the reference problem are defined by Eq.(10).

$$K_I{}^* + iK_{II}{}^* = (T + iS)\sqrt{\pi a}(1 + 2i\varepsilon) \tag{10}$$

Regarding the reference problem in Fig.2, denote $\sigma_{y0,FEM}^{T=1,S=0}*$, $\tau_{xy0,FEM}^{T=1,S=0}*$ are values of stresses for $(T,S) = (1,0)$, and $\sigma_{y0,FEM}^{T=0,S=1}*$ $\tau_{xy0,FEM}^{T=0,S=1}*$ are ones for $(T,S) = (0,1)$. In order to satisfy Eq.(6), stresses at the crack tip of the reference problem are expressed as

$$\begin{aligned} \sigma_{y0,FEM}* &= \sigma_{y0,FEM}^{T=1,S=0}* \times T + \sigma_{y0,FEM}^{T=0,S=1}* \times S, \\ \tau_{xy0,FEM}* &= \tau_{xy0,FEM}^{T=1,S=0}* \times T + \tau_{xy0,FEM}^{T=0,S=1}* \times S \end{aligned} \tag{11}$$

By substituting Eq.(6) into Eq.(11) with T=1, the value of S is obtained as

$$S = \frac{\sigma_{y0,FEM} \times \tau_{xy0,FEM}^{T=1,S=0}* - \tau_{xy0,FEM} \times \sigma_{y0,FEM}^{T=1,S=0}*}{\tau_{xy0,FEM} \times \sigma_{y0,FEM}^{T=0,S=1}* - \sigma_{y0,FEM} \times \tau_{xy0,FEM}^{T=0,S=1}*}. \tag{12}$$

Stress Intensity Factors of an Interface Crack in a Bonded Infinite Plate

To express the results the following dimensionless stress intensity factors F_I , F_{II} are used.

$$K_I + iK_{II} = \left(F_I + iF_{II}\right)(1 + 2i\varepsilon)\sigma\sqrt{\pi a} \tag{13}$$

Dundurs' bi-material parameters α, β are defined in Eq.(14).

$$\alpha = \frac{G_1(\kappa_2 + 1) - G_2(\kappa_1 + 1)}{G_1(\kappa_2 + 1) + G_2(\kappa_1 + 1)},\ \beta = \frac{G_1(\kappa_2 - 1) - G_2(\kappa_1 - 1)}{G_1(\kappa_2 + 1) + G_2(\kappa_1 + 1)} \tag{14}$$

Effect of Plate Dimensions on the Stress Intensity Factors. In order to discuss bonded infinite plates, it is necessary to consider the effect of the plate dimensions on the stress intensity factors because the finite element method cannot treat the infinite plates directly. The results of central interface crack in Fig.1 (a) are therefore investigated in Table 2 with varying a/W = 1/1620, 1/3240, 1/6480 and $\alpha = 0.75,\ \ \beta = 0$, $\alpha = 0.9,\ \ \beta = 0$, $\alpha = 0.75$, $\beta = 0.2$. It is seen that results of $a/W < 1/1620$ coincide each other and may have more than 3 digit accuracy. In other words, Table 2 shows that the results for $a/W = 1/1620$ can be used as the infinite plate $a/W \to 0$ with less than 0.09% error. It is also seen that $F_{II} \to 0$ as $a/W \to 0$ under arbitrary material combination. In the following sections, the results for the bonded infinite plate obtained as shown in Table 1 will be discussed.

Central Interface Crack in a Bonded Infinite Plate under Uni-Axial Tension. Fig.3 shows the results of a central interface crack in a bonded infinite plate under uni-axial tension in the y-direction as shown in Fig.1 (a). In Fig.3, Dundur's parameter β is fixed, and the variations of F_I are depicted with varying parameter α . When material 1 and material 2 are exchanged, Dundur's parameters (α, β) become $(-\alpha, -\beta)$. Then the stress intensity factors (F_I , F_{II}) become (F_I , $-F_{II}$). Therefore all material combinations are considered in the range $\alpha > 0$ in Fig.4(a), the shadow regions which can be expressed by $\alpha(\alpha - 2\beta) < 0$ have no stress singularity at the edge of interface .

In Fig.3, the solid curves show the results of a central interface crack under remote tension $\sigma_y = \sigma$, and the extension lines are predicted by the tendency of the solid curves, as some cases of material combination are difficult to be obtained by the FEM. The dashed line shows the results of that under

Table 2 Dimensionless stress intensity factors of crack in Fig.1 (a) with different.

	a/W	$\alpha = 0.75$ $\beta = 0$	$\alpha = 0.75$ $\beta = 0$	$\alpha = 0.75$ $\beta = 0$
F_I	1/1620	0.93955	0.90859	0.95516
	1/3240	0.93962	0.90883	0.95515
	1/6480	0.93982	0.90943	0.95514
	→0	0.94002	0.91003	0.95513
F_{II}	1/1620	2.21×10^{-4}	2.59×10^{-4}	1.11×10^{-4}
	1/3240	1.10×10^{-4}	1.28×10^{-4}	5.53×10^{-5}
	1/6480	5.51×10^{-5}	6.42×10^{-5}	2.76×10^{-5}
	→0	0	0	0

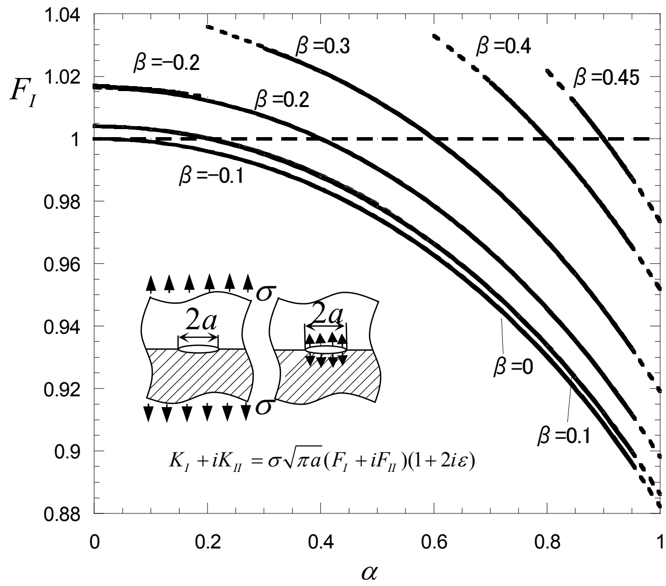
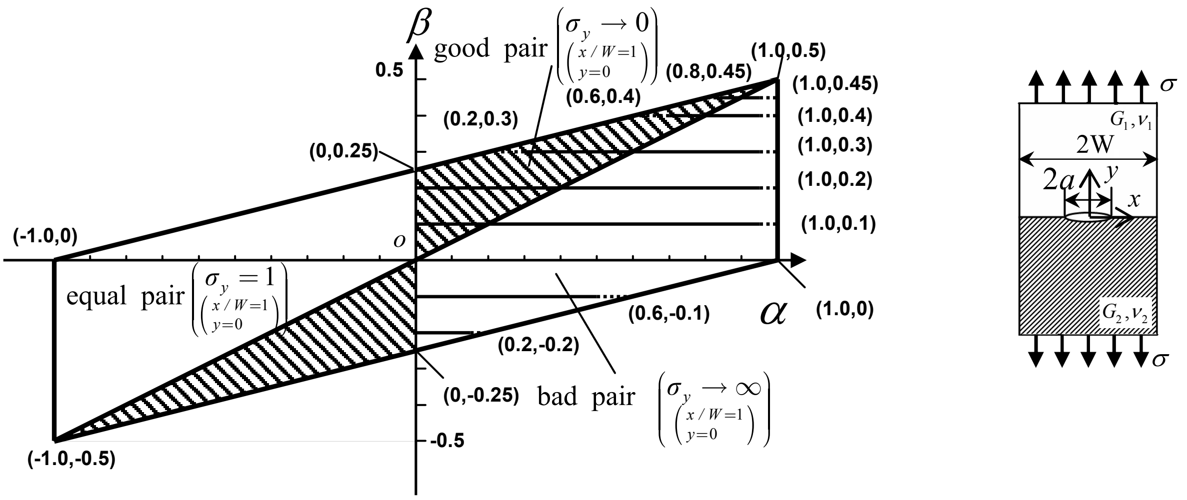
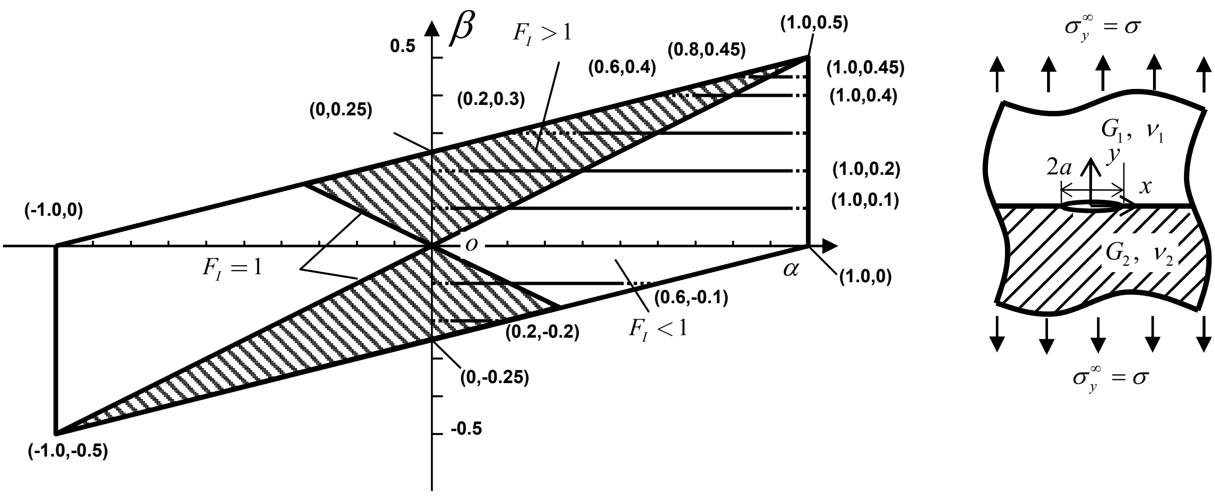


Fig.3 F_1 of a central interface crack in a bonded infinite plate under uni-axal tension, which is corresponding to Fig.1 (a) with $a/W \to 0$.



(a) Shadow regions $\alpha(\alpha-2\beta)<0$ have no stress singular at the edge $x=\pm W$ in Fig.1 (a)



(b) Shadow regions $(\alpha+2\beta)(\alpha-2\beta)<0$ have $F_I>1$ for Fig.1 (a) with $a/W \to 0$

Fig.4 The map of α and β

Table 3 Dimensionless stress intensity factor F_I in Fig.1 (a) with $a/W \to 0$

——— indicates $F_I = 1$ for $(\alpha+2\beta)(\alpha-2\beta)=0$; $F_I > 1$ for $(\alpha+2\beta)(\alpha-2\beta)<0$; $F_I < 1$ for $(\alpha+2\beta)(\alpha-2\beta)>0$

α \ β	−0.2	−0.1	0	0.1	0.2	0.3	0.4	0.45
0.00	1.017	1.004	1.000	1.004	1.017	—	—	—
0.05	1.016	1.004	1.000	1.004	1.016	—	—	—
0.10	1.016	1.003	0.999	1.003	1.015	—	—	—
0.15	1.015	1.002	0.998	1.002	1.014	—	—	—
0.20	(1.013)	1.000	0.996	1.000	1.012	(1.036)	—	—
0.30	—	0.995	0.991	0.995	1.007	1.029	—	—
0.40	—	0.988	0.984	0.988	1.000	1.022	—	—
0.50	—	0.979	0.975	0.978	0.990	1.012	—	—
0.60	—	(0.966)	0.963	0.966	0.979	1.000	(1.033)	—
0.70	—	—	0.948	0.952	0.964	0.985	1.018	—
0.75	—	—	0.940	0.943	0.955	0.977	1.010	—
0.80	—	—	0.930	0.934	0.946	0.967	1.000	(1.022)
0.85	—	—	0.920	0.924	0.935	0.957	0.990	1.011
0.90	—	—	0.910	0.912	0.924	0.945	0.978	1.000
0.95	—	—	0.896	0.900	0.912	0.933	0.965	0.987
1.00	—	—	(0.882)	(0.886)	(0.898)	(0.919)	(0.952)	(0.974)

internal pressure σ whose solution is known as $F_I = 1$ and $F_{II} = 0$. Fig.3 shows the variation of $F_I = 0.882 \sim 1.036$ where the minimum value is $F_I = 0.882$ when $\alpha = 1.0, \beta = 0$, and the maximum value is $F_I = 1.036$ when $\alpha = 0.2, \beta = 0.3$. It is also found that $F_{II} = 0$ for the full range of α, β . Therefore it may be concluded that central interface crack in a bonded infinite plate under remote tension of $\sigma = 1$ is equivalent to that under internal pressure of $\sigma = 0.882 \sim 1.036$. All the values in Fig.3 are given in Table 3 with 3 decimal. The conclusions that $F_I > 1.0$ when $(\alpha+2\beta)(\alpha-2\beta)<0$, $F_I = 1.0$ when $(\alpha+2\beta)(\alpha-2\beta)=0$ and $F_I < 1$ when $(\alpha+2\beta)(\alpha-2\beta)>0$ can be made, and they are shown in the map of α and β Fig.4(b). Fig. 4(b) is similar to Fig.4(a) but they are not exactly the same.

Central Interface Crack in a Bonded Infinite Plate with Material 1 under Tension in the x-direction. Table 4 shows stress intensity factors of central interface crack shown in Fig.1(b) with different relative crack size $a/W = 1/1620$, $1/3240$, $1/6480$ under different material combination $\alpha = 0.3, \beta = 0.2$, $\alpha = -0.75, \beta = 0$, $\alpha = -0.8, \beta = -0.4$. It is seen that all the results coincide each other more than 3 digit when $a/W < 1/1620$.

Fig.5 shows the results of bonded infinite plate $a/W \to 0$ with material 1 under tension in the x-direction, and the extension lines are predicted by the tendency of the solid curves, as some cases of material combination are difficult to be obtained by the FEM. In Fig.5, β in each curve is fixed, and the variations of F_I are depicted with varying parameter α . Previously, it has been thought that tension in the x direction does not contribute to the stress intensity factors [5]. However, as can be seen from Fig.5, F_I is not 0 in current research. It should be noted that the stress intensity factor F_I is not zero under x-directional tension, except if $\varepsilon_{x1} = \varepsilon_{x2}$ is produced along the interface, and it is the minimum value $F_I = -0.034$ appears when $\alpha = 0.2, \beta = 0.3$, and the maximum value $F_I = 0.267$ appears when

Table 4 Dimensionless stress intensity factors of crack in Fig.1 (b) with different a/W .

	a/W	$\alpha = 0.3$, $\beta = 0.2$	$\alpha = -0.75$, $\beta = 0$	$\alpha = -0.8$, $\beta = -0.4$
F_I	1/1620	−0.02540	0.07051	0.17962
	1/3240	−0.02539	0.07056	0.17960
	1/6480	−0.02539	0.07055	0.17959
	→0	−0.02539	0.07054	0.17958
F_{II}	1/1620	1.11×10^{-4}	2.59×10^{-4}	3.28×10^{-4}
	1/3240	5.53×10^{-5}	1.29×10^{-4}	1.64×10^{-4}
	1/6480	2.77×10^{-5}	6.46×10^{-5}	8.19×10^{-5}
	→0	0	0	0

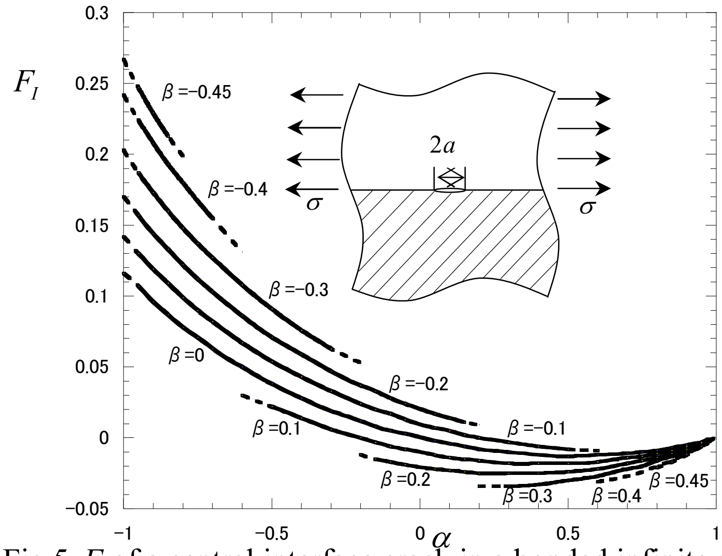


Fig.5 F_{I} of a central interface crack in a bonded infinite plate under $\sigma_{x1}^{\infty} = \sigma$ (see Fig. 1 (b) with $a/W \to 0$)

$\alpha = -1.0, \beta = -0.45$. It is also found that F_{II} =0 for the central interface crack under x-directional tension. Therefore, it may be concluded that central interface crack in a bonded infinite plate with material 1 under x-directional remote tension is equivalent to that of under internal pressure of $\sigma = -0.034 \sim 0.267$.

Conclusions

In this study, stresses at the interface crack tip will be calculated by applying the finite element method. Then the stress intensity factors are determined from the results of the reference problem and given unknown problem. The conclusions are given as following.

1. Stress intensity factors of a central interface crack in bonded infinite plate under remote tension were calculated very accurately under arbitrary material combination.
2. Central interface crack in bonded infinite plate under remote tension of $\sigma = 1$ is equivalent to that under internal pressure of $\sigma = 0.882 \sim 1.036$. Moreover, central interface crack in bonded infinite plate with material 1 under remote x-directional tension of $\sigma = 1$ is equivalent to that under internal pressure of $\sigma = -0.034 \sim 0.267$.

References

[1] K. Oda, K. Kamisugi, and N. A. Noda: Transactions of the Japan Society of Mechanical Engineers, Series A Vol.. 75, (2009), pp. 467-482.
[2] T. Matsumoto, M. Tanaka, and R. Obara: Transactions of the Japan Society of Mechanical Engineers, Series A Vol.65, No.638(1999), pp.2121-2127.
[3] N. Miyazaki, T. Ikeda, T. Soda, and T. Munakata: Transactions of the Japan Society of Mechanical Engineers, Series A Vol.57, No.541(1991), pp. 2063-2069.
[4] T. Teranisi, and H. Nisitani: Transactions of the Japan Society of Mechanical Engineers, Series A, Vol. 65, (1999), pp. 16-21.
[5] Y. Yuki: Mechanics of Interface (1992), p. 102,Baifuukan.

Fracture and Strength of Solids VII

doi:10.4028/www.scientific.net/KEM.462-463

Stress Intensity Factor of a Central Interface Crack in a Bonded Strip under Arbitrary Material Combination

doi:10.4028/www.scientific.net/KEM.462-463.1146