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Single and Double Edge Interface Cracks in a Bonded Plate under Arbitrary Material Combination

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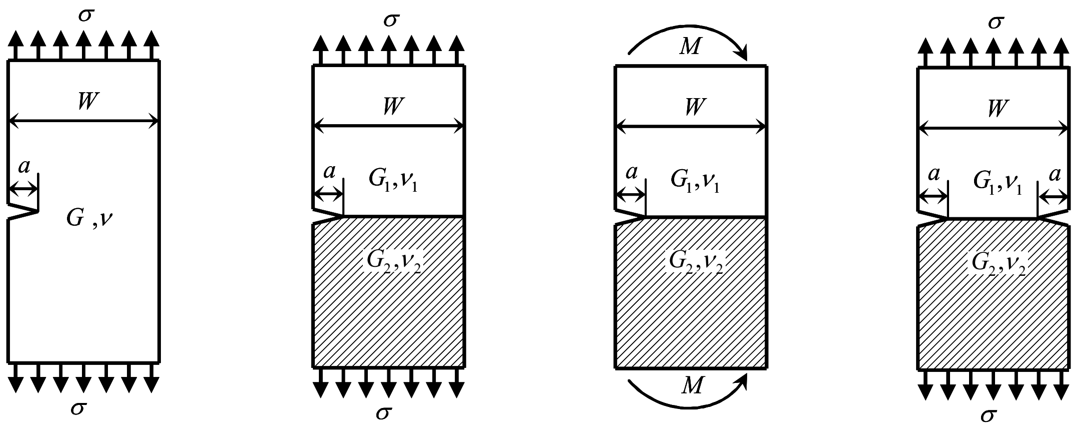
Abstract. Although a lot of interface crack problems were previously treated, few solutions are available under arbitrary material combinations. This paper deals with a single edge interface crack as well as a double edge interface crack in a bonded plate. Then, the effects of material combination on the stress intensity factors are discussed. A useful method to calculate the stress intensity factor of interface crack is presented with focusing on the stresses at the crack tip calculated by the finite element method. Then, the stress intensity factors are indicated in charts under arbitrary material combinations. Specifically, some necessary skills as refined mesh and extrapolations of the stress intensity factors are used to improve the accuracy of the calculation. It has been proved that the values shown in this paper have at least 3-digit accuracy. For the edge interface crack, it is found that the dimensionless stress intensity factors are not always finite depending on Dunders' parameters α, β. For example, they are infinite when $\alpha(\alpha-2\beta)>0$. And they are finite when $\alpha(\alpha-2\beta)=0$, and zero when $\alpha(\alpha-2\beta)<0$.

Introduction

It is widely known that a crack usually initiates at the surface of a homogenous plate, then, gradually grows into inside. Similar phenomenon can be observed for the bonded plate. Therefore, the stress intensity factor of an edge interface crack under arbitrary material combination is dramatically crucial and fundamental. Previously, edge crack in homogenous plate shown in Fig. 1(a) was systematically investigated and solved. However, only limited results concerning several given material combinations are obtained in problems as Fig.1(b) (c) (d). In this paper, edge interface crack problems are investigated with varying material combination as well as the normalized crack size. This provides us not only an easy tool for the in-depth comprehension of the characteristics of 2-dimensional edge interface crack but also a good foundation for the further research.

In this paper, a new method using stresses at the crack tip calculated by FEM is applied for 2-dimensional bi-material bonded plates to obtain the exact stress intensity factors of the edge



(a) (b) (c) (d)

Fig. 1 (a) Edge crack in a homogeneous plate,
Edge interface crack in a bonded plate subjected to (b) tension and (c) bending moment,
(d) Double edge interface crack in a bonded plate

interface crack. Meanwhile, by adopting new mesh type around the crack as well as further refinement and extrapolation, calculation accuracy is significantly improved.

Analysis Method

The zero element method extended to the interface crack problems. An effective method using stresses at the crack tip to calculate the stress intensity factors of an interface crack was recently proposed by ODA et al [1]. If the same mesh size and pattern are applied to the reference and given unknown problems, stress intensity factors can be obtained from the stresses at the crack tip calculated by FEM. Stress intensity factors of the given unknown problem can be obtained by:

$$K_I = \frac{\sigma_{y0,FEM}}{\sigma_{y0,FEM}^{\;*}} K_I^* \quad , \quad K_{II} = \frac{\tau_{xy0,FEM}}{\tau_{xy0,FEM}^{\;*}} K_{II}^* \tag{1}$$

Here, the values with asterisks are those of the reference problem. Stress intensity factors K_I^*, K_{II}^* of the reference problem are defined by Eq. 2.

$$K_I^* + iK_{II}^* = (T + iS)\sqrt{\pi a}(1 + 2i\varepsilon) \tag{2}$$

Here, the reference need to subject to a suitable T and S. Assuming T=1, then, the value of S is obtained by Eq. 3. The detailed information about the zero element method can be found in reference [1].

$$S = \frac{\sigma_{y0,FEM} \times \tau_{xy0,FEM}^{T=1,S=0}{}^* - \tau_{xy0,FEM} \times \sigma_{y0,FEM}^{T=1,S=0}{}^*}{\tau_{xy0,FEM} \times \sigma_{y0,FEM}^{T=0,S=1}{}^* - \sigma_{y0,FEM} \times \tau_{xy0,FEM}^{T=0,S=1}{}^*} \tag{3}$$

It is certificated that the method in Ref. [1] cannot be used into solving deep edge crack problems, so some measures are performed to improve the analysis accuracy in this research. As is known to all, the finite element method treats a problem using finite elements which introduces the calculation error. This error can be neglected in treating center crack problems and non-deep edge crack problems. However, in treating the deep edge crack problems, even if further mesh refinement were taken, calculation accuracy still cannot be significantly improved. This explains why it is impossible to get the accurate values by merely subdivision in the deep edge crack problems. Furthermore, intemperately refinement in the FEM model consumes too much computer resources and makes it impossible for the calculation.

Figure 2 shows the effects of the minimum element size of the FEM model to the stress intensity factors of an edge interface crack $a/W = 0.8$ in a bonded plate. As can be seen from Fig. 2(a), F_I grows linearly with the minimum element size $e_{\min}$. Similarly, linear relationship can also be concluded from Fig2. (b). However, in the case of F_{II}, only if $e_{\min} < a/3^6$, the linear relationship is satisfied. The same conclusions can also be found for the case of an edge interface crack in a bonded plate under bending moment. Therefore, in this research, linear extrapolation of F_I, F_{II} for $e_{\min} = 0$ is employed to calculate the stress intensity factors of the deep edge crack problems. It has

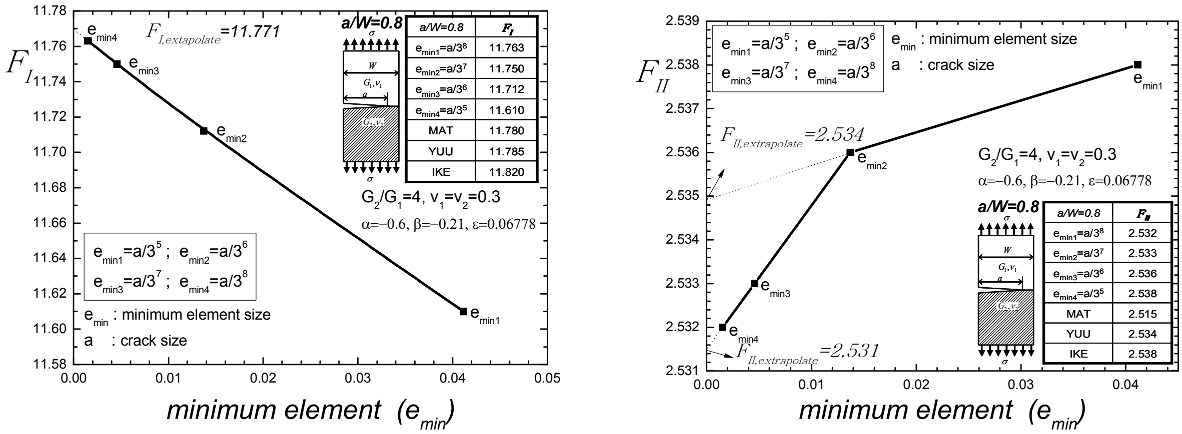


Fig. 2 (a) F_I of edge interface crack $a/W = 0.8$ (b) F_{II} of edge interface crack $a/W = 0.8$

been verified in this paper, when the normalized crack size $a/W < 0.8$, stress intensity factors have 4-digit accuracy, when $a/W \geq 0.8$, it has at least 3-digit accuracy.

Analysis results

Dimensionless stress intensity factors F_I, F_{II} in this research is defined as:

$$K_I + iK_{II} = (F_I + iF_{II})\sigma\sqrt{\pi a}(1+2i\varepsilon) \qquad (4)$$

it is known that stress intensity factor is only determined by Dunders' bi-material combination α,β alone. The definition of α,β is shown in Eq. 5. If material 1 in Fig. 1(b),(c),(d) were substituted by material 2 and material 2 by material 1, α,β would change into $-\alpha,-\beta$ according to Eq. 5. What's more in this case $F_I \to F_I, F_{II} \to -F_{II}$. That means $F_I \to F_I, F_{II} \to -F_{II}$ when $\alpha \to -\alpha$, $\beta \to -\beta$. Therefore, only stress intensity factors under $\alpha \geq 0$ are shown in this research.

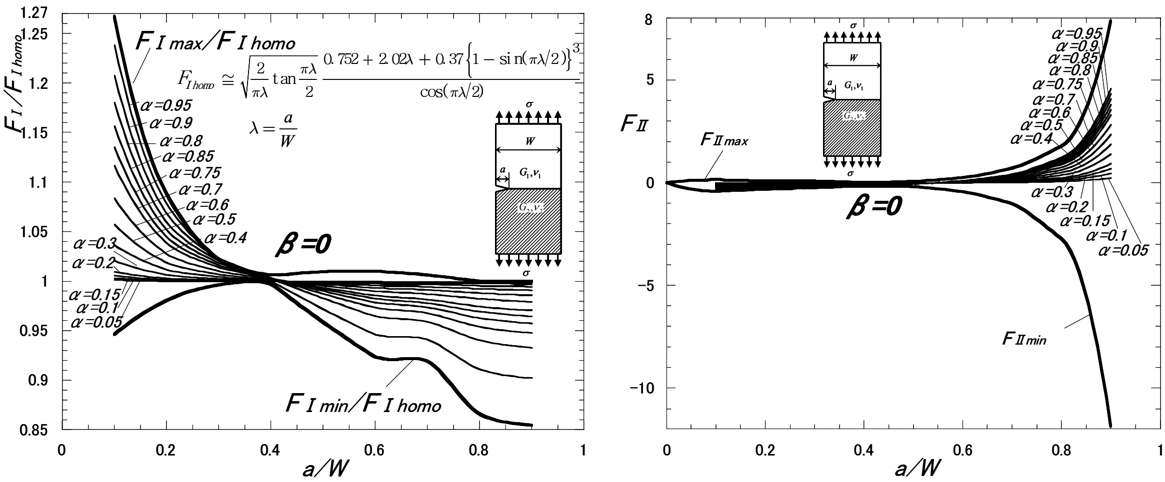
$$\alpha = \frac{G_1(\kappa_2+1) - G_2(\kappa_1+1)}{G_1(\kappa_2+1) + G_2(\kappa_1+1)}, \quad \beta = \frac{G_1(\kappa_2-1) - G_2(\kappa_1-1)}{G_1(\kappa_2+1) + G_2(\kappa_1+1)} \qquad (5)$$

Stress intensity factors of the single edge interface crack in a bonded plate subjected to tension and bending moment. Stress intensity factors of an edge interface crack in a bonded plate subjected to tension and bending moment are systematically investigated with varying the normalized crack size $a/W = 0.1 \sim 0.9$.

Figure 3 shows the variations of the stress intensity factors in a bonded plate under tension with varying the normalized crack size a/W when $\beta = 0$. In Fig. 3(a), the value F_I is normalized using the dimensionless stress intensity factor in a homogenous plate $F_{I\,homo}$ [2,3]. The curves at the top and bottom of each figure show the maximum and minimum stress intensity factors of all material combination respectively. Others show the stress intensity factors of a given α,β. As can be seen from Fig. 3(a), there is a crossing point around a/W=0.4. The ratio $F_I / F_{I\,homo}$ grows with the increase of α before this crossing point, and grows with the decrease of α after this point. However, F_{II} grow monotonically with the increase of α when β is kept constant.

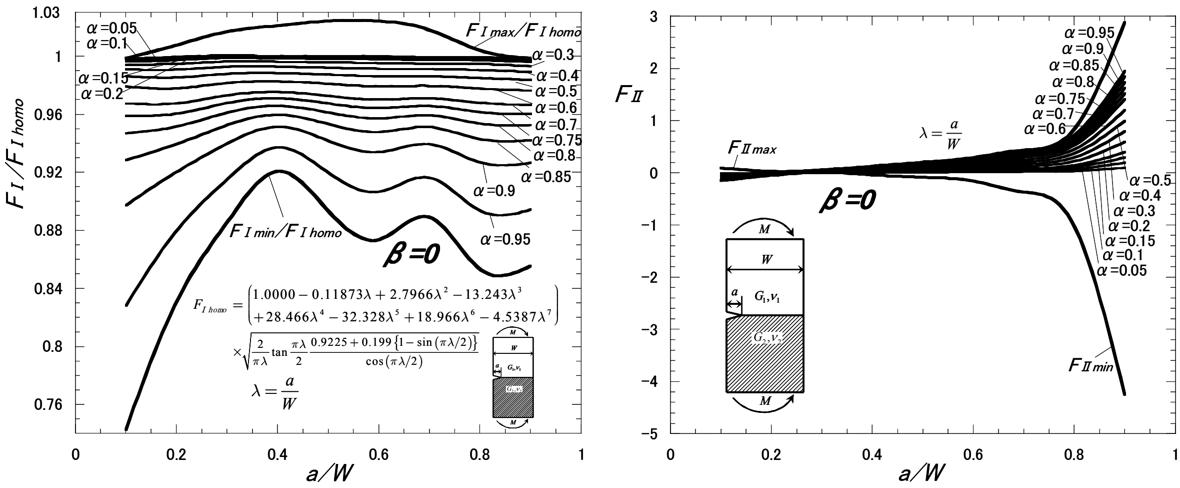
Figure 4 shows the variations of the stress intensity factors in a bonded plate under bending moment with varying the normalized crack size a/W when $\beta = 0$. Similarly, the curves at the top and bottom of Fig. 4 show the maximum and minimum dimensionless stress intensity factors of all material combination ratios respectively. The ratio $F_I / F_{I\,homo}$ increases monotonically with the decrease of α when β is kept constant, and F_{II} grows with the increase of α.

Stress intensity factors of the double edge interface crack in a bonded plate under tension. Figure 5 shows dimensionless stress intensity factors of single and double edge interface cracks under arbitrary material combination when $a/W = 0.1$. It was usually supposed that stress intensity



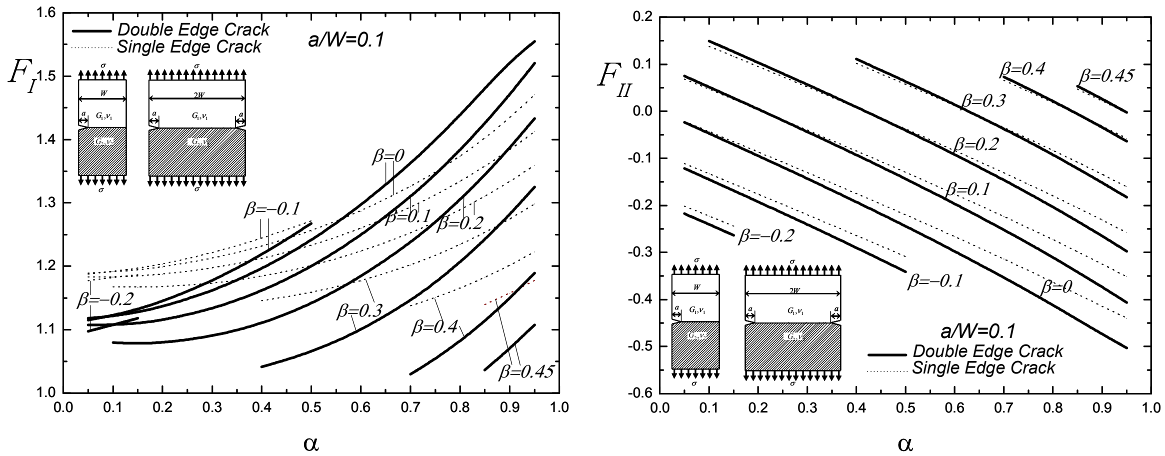
(a) Values of $F_I/F_{I\,homo}$ (b) Values of F_{II}

Fig. 3 Stress intensity factors of edge interface crack in bonded plate under tension



(a) Values of $F_I/F_{I\,\mathrm{homo}}$ (b) Values of F_{II}

Fig. 4 Stress intensity factors of edge interface crack in a bonded plate under bending moment



(a) F_I of edge interface crack $a/W=0.1$ (b) F_{II} of edge interface crack $a/W=0.1$

Fig. 5 Stress intensity factors of single and double edge interface crack in a bonded plate under tension

factors of a single edge interface crack are bigger than that of a double edge crack. However, as can be seen from Fig. 5, conclusions can be made that for specific material combination, F_I and F_{II} of a double edge interface crack are also possibly bigger than that of a single edge crack.

Conlusions

In this paper, by adopting a new mesh type around the crack as well as mesh refinement and linear extrapolation, the zero element method is successfully applied into solving deep crack problems with a good accuracy. Variations of F_I, F_{II} of the edge interface crack in a bonded plate subjected to tension and bending are investigated with α, β and a/W. Meanwhile, stress intensity factors of a double edge interface crack are also discussed in this paper.

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